

Chapter 8—Linear Programming: Sensitivity Analysis and Interpretation of Solution

MULTIPLE CHOICE

1. To solve a linear programming problem with thousands of variables and constraints
 - a. a personal computer can be used.
 - b. a mainframe computer is required.
 - c. the problem must be partitioned into subparts.
 - d. unique software would need to be developed.

ANS: A PTS: 1 TOP: Computer solution

2. A negative dual price for a constraint in a minimization problem means
 - a. as the right-hand side increases, the objective function value will increase.
 - b. as the right-hand side decreases, the objective function value will increase.
 - c. as the right-hand side increases, the objective function value will decrease.
 - d. as the right-hand side decreases, the objective function value will decrease.

ANS: A PTS: 1 TOP: Dual price

3. If a decision variable is not positive in the optimal solution, its reduced cost is
 - a. what its objective function value would need to be before it could become positive.
 - b. the amount its objective function value would need to improve before it could become positive.
 - c. zero.
 - d. its dual price.

ANS: B PTS: 1 TOP: Reduced cost

4. A constraint with a positive slack value
 - a. will have a positive dual price.
 - b. will have a negative dual price.
 - c. will have a dual price of zero.
 - d. has no restrictions for its dual price.

ANS: C PTS: 1 TOP: Slack and dual price

5. The amount by which an objective function coefficient can change before a different set of values for the decision variables becomes optimal is the
 - a. optimal solution.
 - b. dual solution.
 - c. range of optimality.
 - d. range of feasibility.

ANS: C PTS: 1 TOP: Range of optimality

6. The range of feasibility measures
 - a. the right-hand-side values for which the objective function value will not change.
 - b. the right-hand-side values for which the values of the decision variables will not change.
 - c. the right-hand-side values for which the dual prices will not change.
 - d. each of these choices are true.

ANS: C PTS: 1 TOP: Range of feasibility

7. The 100% Rule compares
- proposed changes to allowed changes.
 - new values to original values.
 - objective function changes to right-hand side changes.
 - dual prices to reduced costs.

ANS: A PTS: 1 TOP: Simultaneous changes

8. An objective function reflects the relevant cost of labor hours used in production rather than treating them as a sunk cost. The correct interpretation of the dual price associated with the labor hours constraint is
- the maximum premium (say for overtime) over the normal price that the company would be willing to pay.
 - the upper limit on the total hourly wage the company would pay.
 - the reduction in hours that could be sustained before the solution would change.
 - the number of hours by which the right-hand side can change before there is a change in the solution point.

ANS: A PTS: 1 TOP: Dual price

9. A section of output from The Management Scientist is shown here.

Variable	Lower Limit	Current Value	Upper Limit
1	60	100	120

What will happen to the solution if the objective function coefficient for variable 1 decreases by 20?

- Nothing. The values of the decision variables, the dual prices, and the objective function will all remain the same.
- The value of the objective function will change, but the values of the decision variables and the dual prices will remain the same.
- The same decision variables will be positive, but their values, the objective function value, and the dual prices will change.
- The problem will need to be resolved to find the new optimal solution and dual price.

ANS: B PTS: 1 TOP: Range of optimality

10. A section of output from The Management Scientist is shown here.

Constraint	Lower Limit	Current Value	Upper Limit
2	240	300	420

What will happen if the right-hand-side for constraint 2 increases by 200?

- Nothing. The values of the decision variables, the dual prices, and the objective function will all remain the same.
- The value of the objective function will change, but the values of the decision variables and the dual prices will remain the same.
- The same decision variables will be positive, but their values, the objective function value, and the dual prices will change.
- The problem will need to be resolved to find the new optimal solution and dual price.

ANS: D PTS: 1 TOP: Range of feasibility

11. The amount that the objective function coefficient of a decision variable would have to improve before that variable would have a positive value in the solution is the
- dual price.

- b. surplus variable.
- c. reduced cost.
- d. upper limit.

ANS: C PTS: 1 TOP: Interpretation of computer output

12. The dual price measures, per unit increase in the right hand side,
- a. the increase in the value of the optimal solution.
 - b. the decrease in the value of the optimal solution.
 - c. the improvement in the value of the optimal solution.
 - d. the change in the value of the optimal solution.

ANS: C PTS: 1 TOP: Interpretation of computer output

13. Sensitivity analysis information in computer output is based on the assumption of
- a. no coefficient change.
 - b. one coefficient change.
 - c. two coefficient change.
 - d. all coefficients change.

ANS: B PTS: 1 TOP: Simultaneous changes

14. When the cost of a resource is sunk, then the dual price can be interpreted as the
- a. minimum amount the firm should be willing to pay for one additional unit of the resource.
 - b. maximum amount the firm should be willing to pay for one additional unit of the resource.
 - c. minimum amount the firm should be willing to pay for multiple additional units of the resource.
 - d. maximum amount the firm should be willing to pay for multiple additional units of the resource.

ANS: B PTS: 1 TOP: Dual price

15. The cost that varies depending on the values of the decision variables is a
- a. reduced cost.
 - b. relevant cost.
 - c. sunk cost.
 - d. dual cost.

ANS: B PTS: 1 TOP: Reduced cost

16. Which of the following is not a question answered by standard sensitivity analysis information?
- a. If the right-hand side value of a constraint changes, will the objective function value change?
 - b. Over what range can a constraint's right-hand side value without the constraint's dual price possibly changing?
 - c. By how much will the objective function value change if the right-hand side value of a constraint changes beyond the range of feasibility?
 - d. By how much will the objective function value change if a decision variable's coefficient in the objective function changes within the range of optimality?

ANS: C PTS: 1 TOP: Interpretation of computer output

17. A cost that is incurred no matter what values the decision variables assume is
- a. a reduced cost.
 - b. an optimal cost.
 - c. a sunk cost.

d. a dual cost.

ANS: C PTS: 1 TOP: Sunk and relevant costs

18. Sensitivity analysis is often referred to as
- feasibility testing
 - duality analysis
 - alternative analysis
 - postoptimality analysis

ANS: D PTS: 1 TOP: Introduction

19. Sensitivity analysis is concerned with how certain changes affect
- the feasible solution
 - the unconstrained solution
 - the optimal solution
 - the degenerative solution

ANS: C PTS: 1 TOP: Introduction

20. The dual price for a \leq constraint
- will always be ≤ 0
 - will always be ≥ 0
 - will be ≤ 0 in a minimization problem and ≥ 0 in a maximization problem
 - will always equal 0

ANS: B PTS: 1 TOP: Dual price

TRUE/FALSE

1. Output from a computer package is precise and answers should never be rounded.

ANS: F PTS: 1 TOP: Computer solution

2. The reduced cost for a positive decision variable is 0.

ANS: T PTS: 1 TOP: Reduced cost

3. When the right-hand sides of two constraints are each increased by one unit, the objective function value will be adjusted by the sum of the constraints' dual prices.

ANS: F PTS: 1 TOP: Simultaneous changes

4. If the range of feasibility indicates that the original amount of a resource, which was 20, can increase by 5, then the amount of the resource can increase to 25.

ANS: T PTS: 1 TOP: Range of feasibility

5. The 100% Rule does not imply that the optimal solution will necessarily change if the percentage exceeds 100%.

ANS: T PTS: 1 TOP: Simultaneous changes

6. For any constraint, either its slack/surplus value must be zero or its dual price must be zero.

ANS: T PTS: 1 TOP: Dual price

7. A negative dual price indicates that increasing the right-hand side of the associated constraint would be detrimental to the objective.

ANS: T PTS: 1 TOP: Dual price

8. Decision variables must be clearly defined before constraints can be written.

ANS: T PTS: 1 TOP: Model formulation

9. Decreasing the objective function coefficient of a variable to its lower limit will create a revised problem that is unbounded.

ANS: F PTS: 1 TOP: Range of optimality

10. The dual price for a percentage constraint provides a direct answer to questions about the effect of increases or decreases in that percentage.

ANS: F PTS: 1 TOP: Dual price

11. The dual price associated with a constraint is the improvement in the value of the solution per unit decrease in the right-hand side of the constraint.

ANS: F PTS: 1 TOP: Interpretation of computer output

12. For a minimization problem, a positive dual price indicates the value of the objective function will increase.

ANS: F PTS: 1
TOP: Interpretation of computer output--a second example

13. There is a dual price for every decision variable in a model.

ANS: F PTS: 1 TOP: Interpretation of computer output

14. The amount of a sunk cost will vary depending on the values of the decision variables.

ANS: F PTS: 1 TOP: Cautionary note on the interpretation of dual prices

15. If the optimal value of a decision variable is zero and its reduced cost is zero, this indicates that alternative optimal solutions exist.

ANS: T PTS: 1 TOP: Interpretation of computer output

16. Any change to the objective function coefficient of a variable that is positive in the optimal solution will change the optimal solution.

ANS: F PTS: 1 TOP: Range of optimality

17. Relevant costs should be reflected in the objective function, but sunk costs should not.

ANS: T PTS: 1 TOP: Cautionary note on the interpretation of dual prices

18. If the range of feasibility for b_1 is between 16 and 37, then if $b_1 = 22$ the optimal solution will not change from the original optimal solution.

ANS: F PTS: 1 TOP: Right-hand sides

19. The 100 percent rule can be applied to changes in both objective function coefficients and right-hand sides at the same time.

ANS: F PTS: 1 TOP: Simultaneous changes

20. If the dual price for the right-hand side of a \leq constraint is zero, there is no upper limit on its range of feasibility.

ANS: T PTS: 1 TOP: Right-hand sides

SHORT ANSWER

1. Describe each of the sections of output that come from The Management Scientist and how you would use each.

ANS:
Answer not provided.

PTS: 1 TOP: Interpretation of computer output

2. Explain the connection between reduced costs and the range of optimality, and between dual prices and the range of feasibility.

ANS:
Answer not provided.

PTS: 1 TOP: Interpretation of computer output

3. Explain the two interpretations of dual prices based on the accounting assumptions made in calculating the objective function coefficients.

ANS:
Answer not provided.

PTS: 1 TOP: Dual price

4. How can the interpretation of dual prices help provide an economic justification for new technology?

ANS:
Answer not provided.

PTS: 1 TOP: Dual price

5. How is sensitivity analysis used in linear programming? Given an example of what type of questions that can be answered.

ANS:

Answer not provided.

PTS: 1 TOP: Sensitivity analysis

6. How would sensitivity analysis of a linear program be undertaken if one wishes to consider simultaneous changes for both the right-hand-side values and objective function.

ANS:

Answer not provided.

PTS: 1 TOP: Simultaneous sensitivity analysis

PROBLEM

1. In a linear programming problem, the binding constraints for the optimal solution are

$$5x + 3y \leq 30$$

$$2x + 5y \leq 20$$

- a. Fill in the blanks in the following sentence:

As long as the slope of the objective function stays between _____ and _____, the current optimal solution point will remain optimal.

- b. Which of these objective functions will lead to the same optimal solution?

1) $2x + 1y$ 2) $7x + 8y$ 3) $80x + 60y$ 4) $25x + 35y$

ANS:

a. $-5/3$ and $-2/5$

b. Objective functions 2), 3), and 4)

PTS: 1 TOP: Graphical sensitivity analysis

2. The optimal solution of the linear programming problem is at the intersection of constraints 1 and 2.

$$\text{Max } 2x_1 + x_2$$

$$\text{s.t. } 4x_1 + 1x_2 \leq 400$$

$$4x_1 + 3x_2 \leq 600$$

$$1x_1 + 2x_2 \leq 300$$

$$x_1, x_2 \geq 0$$

- a. Over what range can the coefficient of x_1 vary before the current solution is no longer optimal?

- b. Over what range can the coefficient of x_2 vary before the current solution is no longer optimal?

- c. Compute the dual prices for the three constraints.

ANS:

a. $1.33 \leq c_1 \leq 4$

b. $.5 \leq c_2 \leq 1.5$

c. Dual prices are .25, .25, 0

PTS: 1

TOP: Graphical sensitivity analysis

3. The binding constraints for this problem are the first and second.

Min $x_1 + 2x_2$

s.t. $x_1 + x_2 \geq 300$
 $2x_1 + x_2 \geq 400$
 $2x_1 + 5x_2 \leq 750$
 $x_1, x_2 \geq 0$

- Keeping c_2 fixed at 2, over what range can c_1 vary before there is a change in the optimal solution point?
- Keeping c_1 fixed at 1, over what range can c_2 vary before there is a change in the optimal solution point?
- If the objective function becomes $\text{Min } 1.5x_1 + 2x_2$, what will be the optimal values of x_1 , x_2 , and the objective function?
- If the objective function becomes $\text{Min } 7x_1 + 6x_2$, what constraints will be binding?
- Find the dual price for each constraint in the original problem.

ANS:

- $.8 \leq c_1 \leq 2$
- $1 \leq c_2 \leq 2.5$
- $x_1 = 250, x_2 = 50, \text{obj. func.} = 475$
- Constraints 1 and 2 will be binding.
- Dual prices are .33, 0, .33 (The first and third values are negative.)

PTS: 1

TOP: Graphical sensitivity analysis

4. Excel's Solver tool has been used in the spreadsheet below to solve a linear programming problem with a maximization objective function and all \leq constraints.

Input Section				
	Objective Function Coefficients			
		<i>x</i>	<i>y</i>	
		4	6	
	Constraints			Avail.
	#1	3	5	60
	#2	3	2	48
	#3	1	1	20
Output Section				
	Variables	13.333333	4	
	Profit	53.333333	24	77.333333
	Constraint	Usage	Slack	
	#1	60	1.789E-11	

	#2	48	-2.69E-11	
	#3	17.333333	2.6666667	

- Give the original linear programming problem.
- Give the complete optimal solution.

ANS:

a. Max $4x + 6y$

s.t. $3x + 5y \leq 60$
 $3x + 2y \leq 48$
 $1x + 1y \leq 20$
 $x, y \geq 0$

- b. The complete optimal solution is $x = 13.333, y = 4, s_1 = 0, s_2 = 0, s_3 = 2.667$, obj. func. = 73.333

PTS: 1 TOP: Spreadsheet solution of LPs

5. Excel's Solver tool has been used in the spreadsheet below to solve a linear programming problem with a minimization objective function and all \geq constraints.

Input Section				
Objective Function Coefficients				
		<i>x</i>	<i>y</i>	
		5	4	
Constraints				Req'd
	#1	4	3	60
	#2	2	5	50
	#3	9	8	144
Output Section				
Variables		9.6	7.2	
Profit		48	28.8	76.8
Constraint		Usage	Slack	
	#1	60	1.35E-11	
	#2	55.2	-5.2	
	#3	144	-2.62E-11	

- Give the original linear programming problem.
- Give the complete optimal solution.

ANS:

a. Min $5x + 4y$

$$\begin{aligned} \text{s.t.} \quad & 4x + 3y \geq 60 \\ & 2x + 5y \geq 50 \\ & 9x + 8y \geq 144 \\ & x, y \geq 0 \end{aligned}$$

b. The complete optimal solution is $x = 9.6, y = 7.2, s_1 = 0, s_2 = 5.2, s_3 = 0$, obj. func. = 76.8

PTS: 1 TOP: Spreadsheet solution of LPs

6. Use the spreadsheet and Solver sensitivity report to answer these questions.
- What is the cell formula for B12?
 - What is the cell formula for C12?
 - What is the cell formula for D12?
 - What is the cell formula for B15?
 - What is the cell formula for B16?
 - What is the cell formula for B17?
 - What is the optimal value for x_1 ?
 - What is the optimal value for x_2 ?
 - Would you pay \$.50 each for up to 60 more units of resource 1?
 - Is it possible to figure the new objective function value if the profit on product 1 increases by a dollar, or do you have to rerun Solver?

	A	B	C	D	E
1					
2	Input Information				
3		Var. 1	Var. 2	(type)	Avail.
4	Constraint 1	2	5	<	40
5	Constraint 2	3	1	<	30
6	Constraint 3	1	1	>	12
7					
8	Profit	5	4		
9					
10	Output Information				
11	Variables				
12	Profit				= Total
13					
14	Resources	Used	Slk/Surp		
15	Constraint 1				
16	Constraint 2				
17	Constraint 3				
18					
19					

Microsoft Excel 7.0 Sensitivity Report
Worksheet: [x3s7base.xls]Sheet1

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
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\$B\$12	Variables Variable	8.461538462	0	5	7	3.4
	1					
\$C\$12	Variables Variable	4.615384615	0	4	8.5	2.333333333
	2					
Constraints						
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$15	constraint 1 Used	40	0.538461538	40	110	7
\$B\$16	constraint 2 Used	30	1.307692308	30	30	4.666666667
\$B\$17	constraint 3 Used	13.07692308	0	12	1.076923077	1E+30

ANS:

- $=B8*B11$
- $=C8*C11$
- $=B12+C12$
- $=B4*B11+C4*C11$
- $=B5*B11+C5*C11$
- $=B6*B11+C6*C11$
- 8.46
- 4.61
- yes
- no

PTS: 1 TOP: Spreadsheet solution of LPs

7. Use the following Management Scientist output to answer the questions.

LINEAR PROGRAMMING PROBLEM

MAX $31X_1+35X_2+32X_3$

S.T.

- $3X_1+5X_2+2X_3>90$
- $6X_1+7X_2+8X_3<150$
- $5X_1+3X_2+3X_3<120$

OPTIMAL SOLUTION

Objective Function Value = 763.333

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X1	13.333	0.000
X2	10.000	0.000
X3	0.000	10.889

<u>Constraint</u>	<u>Slack/Surplus</u>	<u>Dual Price</u>
1	0.000	-0.778
2	0.000	5.556
3	23.333	0.000

OBJECTIVE COEFFICIENT RANGES

<u>Variable</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
X1	30.000	31.000	No Upper Limit
X2	No Lower Limit	35.000	36.167
X3	No Lower Limit	32.000	42.889

RIGHT HAND SIDE RANGES

<u>Constraint</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
1	77.647	90.000	107.143
2	126.000	150.000	163.125
3	96.667	120.000	No Upper Limit

- Give the solution to the problem.
- Which constraints are binding?
- What would happen if the coefficient of x_1 increased by 3?
- What would happen if the right-hand side of constraint 1 increased by 10?

ANS:

- $x_1 = 13.33, x_2 = 10, x_3 = 0, s_1 = 0, s_2 = 0, s_3 = 23.33, \text{obj. func.} = 763.33$
- Constraints 1 and 2 are binding.
- The value of the objective function would increase by 40.
- The value of the objective function would decrease by 7.78.

PTS: 1 TOP: Interpretation of Management Scientist output

- Use the following Management Scientist output to answer the questions.

MIN $4X_1 + 5X_2 + 6X_3$

S.T.

- $X_1 + X_2 + X_3 < 85$
- $3X_1 + 4X_2 + 2X_3 > 280$
- $2X_1 + 4X_2 + 4X_3 > 320$

Objective Function Value = 400.000

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X1	0.000	1.500
X2	80.000	0.000
X3	0.000	1.000

<u>Constraint</u>	<u>Slack/Surplus</u>	<u>Dual Price</u>
1	5.000	0.000
2	40.000	0.000
3	0.000	-1.250

OBJECTIVE COEFFICIENT RANGES

<u>Variable</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
X1	2.500	4.000	No Upper Limit
X2	0.000	5.000	6.000

X3	5.000	6.000	No Upper Limit
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RIGHT HAND SIDE RANGES

<u>Constraint</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
1	80.000	85.000	No Upper Limit
2	No Lower Limit	280.000	320.000
3	280.000	320.000	340.000

- What is the optimal solution, and what is the value of the profit contribution?
- Which constraints are binding?
- What are the dual prices for each resource? Interpret.
- Compute and interpret the ranges of optimality.
- Compute and interpret the ranges of feasibility.

ANS:

- $x_1 = 0, x_2 = 80, x_3 = 0, s_1 = 5, s_2 = 40, s_3 = 0, \text{obj. func.} = 400$
- Constraint 3 is binding.
- Dual prices are 0, 0, and -1.25 .
They measure the improvement in the obj. func. per unit increase in each right-hand side.
- $2.5 \leq c_1 < \infty$
 $0 \leq c_2 \leq 6$
 $5 \leq c_3 < \infty$

As long as the objective function coefficient stays within its range, the current optimal solution point will not change, although obj. func. value could.

- $80 \leq b_1 < \infty$
 $-\infty < b_2 \leq 320$
 $280 \leq b_3 \leq 340$

As long as the right-hand side value stays within its range, the currently binding constraints will remain so, although the values of the decision variables could change. The dual variable values will remain the same.

PTS: 1 TOP: Interpretation of Management Scientist output

- The following linear programming problem has been solved by The Management Scientist. Use the output to answer the questions.

LINEAR PROGRAMMING PROBLEM

$$\text{MAX } 25X_1 + 30X_2 + 15X_3$$

S.T.

- $4X_1 + 5X_2 + 8X_3 < 1200$
- $9X_1 + 15X_2 + 3X_3 < 1500$

OPTIMAL SOLUTION

Objective Function Value = 4700.000

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
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X1	140.000	0.000
X2	0.000	10.000
X3	80.000	0.000

<u>Constraint</u>	<u>Slack/Surplus</u>	<u>Dual Price</u>
1	0.000	1.000
2	0.000	2.333

OBJECTIVE COEFFICIENT RANGES

<u>Variable</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
X1	19.286	25.000	45.000
X2	No Lower Limit	30.000	40.000
X3	8.333	15.000	50.000

RIGHT HAND SIDE RANGES

<u>Constraint</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
1	666.667	1200.000	4000.000
2	450.000	1500.000	2700.000

- Give the complete optimal solution.
- Which constraints are binding?
- What is the dual price for the second constraint? What interpretation does this have?
- Over what range can the objective function coefficient of x_2 vary before a new solution point becomes optimal?
- By how much can the amount of resource 2 decrease before the dual price will change?
- What would happen if the first constraint's right-hand side increased by 700 and the second's decreased by 350?

ANS:

- $x_1 = 140, x_2 = 0, x_3 = 80, s_1 = 0, s_2 = 0, \text{obj. func.} = 4700$
- Constraints 1 and 2 are binding.
- Dual price 2 = 2.33. A unit increase in the right-hand side of constraint 2 will increase the value of the objective function by 2.33.
- As long as $c_2 \leq 40$, the solution will be unchanged.
- 1050
- The sum of percentage changes is $700/2800 + (-350)/(-1050) < 1$ so the solution will not change.

PTS: 1

TOP: Interpretation of Management Scientist output

10. LINGO output is given for the following linear programming problem.

$$\text{MIN } 12 X1 + 10 X2 + 9 X3$$

SUBJECT TO

$$2) \ 5 X1 + 8 X2 + 5 X3 \geq 60$$

$$3) \ 8 X1 + 10 X2 + 5 X3 \geq 80$$

END

LP OPTIMUM FOUND AT STEP 1

OBJECTIVE FUNCTION VALUE

1) 80.000000

<u>VARIABLE</u>	<u>VALUE</u>	<u>REDUCED COST</u>
X1	.000000	4.000000
X2	8.000000	.000000
X3	.000000	4.000000

<u>ROW</u>	<u>SLACK OR SURPLUS</u>	<u>DUAL PRICE</u>
2)	4.000000	.000000
3)	.000000	-1.000000

NO. ITERATIONS= 1

RANGES IN WHICH THE BASIS IS UNCHANGED:

<u>VARIABLE</u>	<u>CURRENT COEFFICIENT</u>	<u>OBJ. COEFFICIENT RANGES</u>	
		<u>ALLOWABLE INCREASE</u>	<u>ALLOWABLE DECREASE</u>
X1	12.000000	INFINITY	4.000000
X2	10.000000	5.000000	10.000000
X3	9.000000	INFINITY	4.000000

<u>ROW</u>	<u>CURRENT RHS</u>	<u>RIGHTHAND SIDE RANGES</u>	
		<u>ALLOWABLE INCREASE</u>	<u>ALLOWABLE DECREASE</u>
2	60.000000	4.000000	INFINITY
3	80.000000	INFINITY	5.000000

- What is the solution to the problem?
- Which constraints are binding?
- Interpret the reduced cost for x_1 .
- Interpret the dual price for constraint 2.
- What would happen if the cost of x_1 dropped to 10 and the cost of x_2 increased to 12?

ANS:

- $x_1 = 0, x_2 = 8, x_3 = 0, s_1 = 4, s_2 = 0$, obj. func. = 80
- Constraint 2 is binding.
- c_1 would have to decrease by 4 or more for x_1 to become positive.
- Increasing the right-hand side by 1 will cause a negative improvement, or increase, of 1 in this minimization objective function.
- The sum of the percentage changes is $(-2)/(-4) + 2/5 < 1$ so the solution would not change.

PTS: 1

TOP: Interpretation of LINGO output

11. The LP problem whose output follows determines how many necklaces, bracelets, rings, and earrings a jewelry store should stock. The objective function measures profit; it is assumed that every piece stocked will be sold. Constraint 1 measures display space in units, constraint 2 measures time to set up the display in minutes. Constraints 3 and 4 are marketing restrictions.

LINEAR PROGRAMMING PROBLEM

$$\text{MAX } 100X_1 + 120X_2 + 150X_3 + 125X_4$$

S.T.

- 1) $X_1 + 2X_2 + 2X_3 + 2X_4 < 108$
- 2) $3X_1 + 5X_2 + X_4 < 120$
- 3) $X_1 + X_3 < 25$
- 4) $X_2 + X_3 + X_4 > 50$

OPTIMAL SOLUTION

Objective Function Value = 7475.000

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X1	8.000	0.000
X2	0.000	5.000
X3	17.000	0.000
X4	33.000	0.000

<u>Constraint</u>	<u>Slack/Surplus</u>	<u>Dual Price</u>
1	0.000	75.000
2	63.000	0.000
3	0.000	25.000
4	0.000	-25.000

OBJECTIVE COEFFICIENT RANGES

<u>Variable</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
X1	87.500	100.000	No Upper Limit
X2	No Lower Limit	120.000	125.000
X3	125.000	150.000	162.500
X4	120.000	125.000	150.000

RIGHT HAND SIDE RANGES

<u>Constraint</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
1	100.000	108.000	123.750
2	57.000	120.000	No Upper Limit
3	8.000	25.000	58.000
4	41.500	50.000	54.000

Use the output to answer the questions.

- a. How many necklaces should be stocked?
- b. Now many bracelets should be stocked?
- c. How many rings should be stocked?
- d. How many earrings should be stocked?
- e. How much space will be left unused?

- f. How much time will be used?
- g. By how much will the second marketing restriction be exceeded?
- h. What is the profit?
- i. To what value can the profit on necklaces drop before the solution would change?
- j. By how much can the profit on rings increase before the solution would change?
- k. By how much can the amount of space decrease before there is a change in the profit?
- l. You are offered the chance to obtain more space. The offer is for 15 units and the total price is 1500. What should you do?

ANS:

- a. 8
- b. 0
- c. 17
- d. 33
- e. 0
- f. 57
- g. 0
- h. 7475
- i. 87.5
- j. 12.5
- k. 0
- l. Say no. Although 15 units can be evaluated, their value (1125) is less than the cost (1500).

PTS: 1

TOP: Interpretation of Management Scientist output

12. The decision variables represent the amounts of ingredients 1, 2, and 3 to put into a blend. The objective function represents profit. The first three constraints measure the usage and availability of resources A, B, and C. The fourth constraint is a minimum requirement for ingredient 3. Use the output to answer these questions.
 - a. How much of ingredient 1 will be put into the blend?
 - b. How much of ingredient 2 will be put into the blend?
 - c. How much of ingredient 3 will be put into the blend?
 - d. How much resource A is used?
 - e. How much resource B will be left unused?
 - f. What will the profit be?
 - g. What will happen to the solution if the profit from ingredient 2 drops to 4?
 - h. What will happen to the solution if the profit from ingredient 3 increases by 1?
 - i. What will happen to the solution if the amount of resource C increases by 2?
 - j. What will happen to the solution if the minimum requirement for ingredient 3 increases to 15?

LINEAR PROGRAMMING PROBLEM

$$\text{MAX } 4X_1 + 6X_2 + 7X_3$$

S.T.

- 1) $3X_1 + 2X_2 + 5X_3 < 120$
- 2) $1X_1 + 3X_2 + 3X_3 < 80$
- 3) $5X_1 + 5X_2 + 8X_3 < 160$
- 4) $+1X_3 > 10$

OPTIMAL SOLUTION

Objective Function Value = 166.000

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X1	0.000	2.000
X2	16.000	0.000
X3	10.000	0.000

<u>Constraint</u>	<u>Slack/Surplus</u>	<u>Dual Price</u>
1	38.000	0.000
2	2.000	0.000
3	0.000	1.200
4	0.000	-2.600

OBJECTIVE COEFFICIENT RANGES

<u>Variable</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
X1	No Lower Limit	4.000	6.000
X2	4.375	6.000	No Upper Limit
X3	No Lower Limit	7.000	9.600

RIGHT HAND SIDE RANGES

<u>Constraint</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
1	82.000	120.000	No Upper Limit
2	78.000	80.000	No Upper Limit
3	80.000	160.000	163.333
4	8.889	10.000	20.000

ANS:

- a. 0
- b. 16
- c. 10
- d. 44
- e. 2
- f. 166
- g. rerun
- h. $Z = 176$
- i. $Z = 168.4$
- j. $Z = 153$

PTS: 1

TOP: Interpretation of Management Scientist output

13. The LP model and LINGO output represent a problem whose solution will tell a specialty retailer how many of four different styles of umbrellas to stock in order to maximize profit. It is assumed that every one stocked will be sold. The variables measure the number of women's, golf, men's, and folding umbrellas, respectively. The constraints measure storage space in units, special display racks, demand, and a marketing restriction, respectively.

$$\text{MAX } 4 X_1 + 6 X_2 + 5 X_3 + 3.5 X_4$$

SUBJECT TO

- 2) $2 X_1 + 3 X_2 + 3 X_3 + X_4 \leq 120$
- 3) $1.5 X_1 + 2 X_2 \leq 54$
- 4) $2 X_2 + X_3 + X_4 \leq 72$
- 5) $X_2 + X_3 \geq 12$

END

OBJECTIVE FUNCTION VALUE

- 1) 318.00000

<u>VARIABLE</u>	<u>VALUE</u>	<u>REDUCED COST</u>
X1	12.000000	.000000
X2	.000000	.500000
X3	12.000000	.000000
X4	60.000000	.000000

<u>ROW</u>	<u>SLACK OR SURPLUS</u>	<u>DUAL PRICE</u>
2)	.000000	2.000000
3)	36.000000	.000000
4)	.000000	1.500000
5)	.000000	-2.500000

RANGES IN WHICH THE BASIS IS UNCHANGED:

<u>VARIABLE</u>	<u>CURRENT</u> <u>COEFFICIENT</u>	<u>OBJ. COEFFICIENT RANGES</u>	
		<u>ALLOWABLE</u> <u>INCREASE</u>	<u>ALLOWABLE</u> <u>DECREASE</u>
X1	4.000000	1.000000	2.500000
X2	6.000000	.500000	INFINITY
X3	5.000000	2.500000	.500000
X4	3.500000	INFINITY	.500000

<u>ROW</u>	<u>CURRENT</u> <u>RHS</u>	<u>RIGHTHAND SIDE RANGES</u>	
		<u>ALLOWABLE</u> <u>INCREASE</u>	<u>ALLOWABLE</u> <u>DECREASE</u>
2	120.000000	48.000000	24.000000
3	54.000000	INFINITY	36.000000
4	72.000000	24.000000	48.000000
5	12.000000	12.000000	12.000000

Use the output to answer the questions.

- a. How many women's umbrellas should be stocked?
- b. How many golf umbrellas should be stocked?
- c. How many men's umbrellas should be stocked?
- d. How many folding umbrellas should be stocked?
- e. How much space is left unused?
- f. How many racks are used?
- g. By how much is the marketing restriction exceeded?
- h. What is the total profit?
- i. By how much can the profit on women's umbrellas increase before the solution would change?
- j. To what value can the profit on golf umbrellas increase before the solution would change?

- k. By how much can the amount of space increase before there is a change in the dual price?
- l. You are offered an advertisement that should increase the demand constraint from 72 to 86 for a total cost of \$20. Would you say yes or no?

ANS:

- a. 12
- b. 0
- c. 12
- d. 60
- e. 0
- f. 18
- g. 0
- h. 318
- i. 1
- j. 6.5
- k. 48
- l. Yes. The dual price is 1.5 for 24 additional units. The value of the ad $(14)(1.5)=21$ exceeds the cost of 20.

PTS: 1 TOP: Interpretation of LINGO output

14. Eight of the entries have been deleted from the LINGO output that follows. Use what you know about linear programming to find values for the blanks.

$$\text{MIN } 6 X_1 + 7.5 X_2 + 10 X_3$$

SUBJECT TO

$$2) \quad 25 X_1 + 35 X_2 + 30 X_3 \geq 2400$$

$$3) \quad 2 X_1 + 4 X_2 + 8 X_3 \geq 400$$

END

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

$$1) \quad 612.50000$$

<u>VARIABLE</u>	<u>VALUE</u>	<u>REDUCED COST</u>
X1	_____	1.312500
X2	_____	_____
X3	27.500000	_____

<u>ROW</u>	<u>SLACK OR SURPLUS</u>	<u>DUAL PRICE</u>
2)	_____	-.125000
3)	_____	-.781250

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ. COEFFICIENT RANGES

<u>VARIABLE</u>	<u>CURRENT COEFFICIENT</u>	<u>ALLOWABLE INCREASE</u>	<u>ALLOWABLE DECREASE</u>
X1	6.000000		
X2	7.500000	1.500000	2.500000
X3	10.000000	5.000000	3.571429

<u>ROW</u>	<u>RIGHTHAND SIDE RANGES</u>		
	<u>CURRENT RHS</u>	<u>ALLOWABLE INCREASE</u>	<u>ALLOWABLE DECREASE</u>
2	2400.000000	1100.000000	900.000000
3	400.000000	240.000000	125.714300

ANS:

It is easiest to calculate the values in this order.

$x_1 = 0$, $x_2 = 45$, reduced cost 2 = 0, reduced cost 3 = 0, row 2 slack = 0, row 3 slack = 0, c_1 allowable decrease = 1.3125, allowable increase = infinity

PTS: 1 TOP: Interpretation of LINGO output

15. Portions of a Management Scientist output are shown below. Use what you know about the solution of linear programs to fill in the ten blanks.

LINEAR PROGRAMMING PROBLEM

MAX $12X_1 + 9X_2 + 7X_3$

S.T.

- 1) $3X_1 + 5X_2 + 4X_3 < 150$
- 2) $2X_1 + 1X_2 + 1X_3 < 64$
- 3) $1X_1 + 2X_2 + 1X_3 < 80$
- 4) $2X_1 + 4X_2 + 3X_3 > 116$

OPTIMAL SOLUTION

Objective Function Value = 336.000

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X1		0.000
X2	24.000	
X3		3.500

<u>Constraint</u>	<u>Slack/Surplus</u>	<u>Dual Price</u>
1	0.000	15.000
2		0.000
3		0.000
4	0.000	

OBJECTIVE COEFFICIENT RANGES

<u>Variable</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
X1	5.400	12.000	No Upper Limit
X2	2.000	9.000	20.000
X3	No Lower Limit	7.000	10.500

RIGHT HAND SIDE RANGES

<u>Constraint</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
1	145.000	150.000	156.667
2	_____	_____	64.000
3	_____	_____	80.000
4	110.286	116.000	120.000

ANS:

$x_3 = 0$ because the reduced cost is positive.

$x_1 = 24$ after plugging into the objective function

The second reduced cost is 0.

$s_2 = 20$ and $s_3 = 22$ from plugging into the constraints.

The fourth dual price is -16.5 from plugging into the dual objective function, which your students might not understand fully until Chapter 6.

The lower limit for constraint 2 is 44 and for constraint 3 is 58, from the amount of slack in each constraint. There are no upper limits for these constraints.

PTS: 1

TOP: Interpretation of Management Scientist output

16. A large sporting goods store is placing an order for bicycles with its supplier. Four models can be ordered: the adult Open Trail, the adult Cityscape, the girl's Sea Sprite, and the boy's Trail Blazer. It is assumed that every bike ordered will be sold, and their profits, respectively, are 30, 25, 22, and 20. The LP model should maximize profit. There are several conditions that the store needs to worry about. One of these is space to hold the inventory. An adult's bike needs two feet, but a child's bike needs only one foot. The store has 500 feet of space. There are 1200 hours of assembly time available. The child's bike need 4 hours of assembly time; the Open Trail needs 5 hours and the Cityscape needs 6 hours. The store would like to place an order for at least 275 bikes.
- Formulate a model for this problem.
 - Solve your model with any computer package available to you.
 - How many of each kind of bike should be ordered and what will the profit be?
 - What would the profit be if the store had 100 more feet of storage space?
 - If the profit on the Cityscape increases to \$35, will any of the Cityscape bikes be ordered?
 - Over what range of assembly hours is the dual price applicable?
 - If we require 5 more bikes in inventory, what will happen to the value of the optimal solution?
 - Which resource should the company work to increase, inventory space or assembly time?

ANS:

NOTE TO INSTRUCTOR: The problem is suitable for a take-home or lab exam. The student must formulate the model, solve the problem with a computer package, and then interpret the solution to answer the questions.

a. $\text{MAX } 30 X_1 + 25 X_2 + 22 X_3 + 20 X_4$

SUBJECT TO

2) $2 X_1 + 2 X_2 + X_3 + X_4 \leq 500$

3) $5 X_1 + 6 X_2 + 4 X_3 + 4 X_4 \leq 1200$

4) $X_1 + X_2 + X_3 + X_4 \geq 275$

b. OBJECTIVE FUNCTION VALUE

1) 6850.0000

<u>VARIABLE</u>	<u>VALUE</u>	<u>REDUCED COST</u>
X1	100.000000	.000000
X2	.000000	13.000000
X3	175.000000	.000000
X4	.000000	2.000000

<u>ROW</u>	<u>SLACK OR SURPLUS</u>	<u>DUAL PRICE</u>
2)	125.000000	.000000
3)	.000000	8.000000
4)	.000000	-10.000000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

<u>VARIABLE</u>	<u>CURRENT COEFFICIENT</u>	<u>OBJ. COEFFICIENT RANGES</u>	
		<u>ALLOWABLE INCREASE</u>	<u>ALLOWABLE DECREASE</u>
X1	30.000000	INFINITY	2.500000
X2	25.000000	13.000000	INFINITY
X3	22.000000	2.000000	2.000000
X4	20.000000	2.000000	INFINITY

<u>ROW</u>	<u>CURRENT RHS</u>	<u>RIGHTHAND SIDE RANGES</u>	
		<u>ALLOWABLE INCREASE</u>	<u>ALLOWABLE DECREASE</u>
2	500.000000	INFINITY	125.000000
3	1200.000000	125.000000	100.000000
4	275.000000	25.000000	35.000000

- c. Order 100 Open Trails, 0 Cityscapes, 175 Sea Sprites, and 0 Trail Blazers. Profit will be 6850.
- d. 6850
- e. No. The \$10 increase is below the reduced cost.
- f. 1100 to 1325
- g. It will decrease by 50.
- h. Assembly time.

PTS: 1 TOP: Formulation and computer solution

17. A company produces two products made from aluminum and copper. The table below gives the unit requirements, the unit production man-hours required, the unit profit and the availability of the resources (in tons).

	<u>Aluminum</u>	<u>Copper</u>	<u>Man-hours</u>	<u>Unit Profit</u>
Product 1	1	0	2	50
Product 2	1	1	3	60
Available	10	6	24	

The Management Scientist provided the following solution output:

Objective Function Value = 540.000

<u>VARIABLE</u>	<u>VALUE</u>	<u>REDUCED COST</u>
X1	6.000	0.000
X2	4.000	0.000

<u>CONSTRAINT</u>	<u>SLACK/SURPLUS</u>	<u>DUAL PRICE</u>
1	.000	30.000
2	2.000	0.000
3	0.000	10.000

RANGES IN WHICH THE BASIS IS UNCHANGED:

<u>VARIABLE</u>	<u>CURRENT COEFFICIENT</u>	<u>OBJ. COEFFICIENT RANGES</u>	
		<u>ALLOWABLE INCREASE</u>	<u>ALLOWABLE DECREASE</u>
X1	50.000	10.000	10.000
X2	60.000	15.000	10.000

<u>CONSTRAINT</u>	<u>CURRENT RHS</u>	<u>RIGHTHAND SIDE RANGES</u>	
		<u>ALLOWABLE INCREASE</u>	<u>ALLOWABLE DECREASE</u>
1	10.000	2.000	1.000
2	6.000	INFINITY	2.000
3	24.000	2.000	4.000

- What is the optimal production schedule?
- Within what range for the profit on product 2 will the solution in (a) remain optimal? What is the optimal profit when $c_2 = 70$?
- Suppose that simultaneously the unit profits on x_1 and x_2 changed from 50 to 55 and 60 to 65 respectively. Would the optimal solution change?
- Explain the meaning of the "DUAL PRICES" column. Given the optimal solution, why should the dual price for copper be 0?
- What is the increase in the value of the objective function for an extra unit of aluminum?
- Man-hours were not figured into the unit profit as it must pay three workers for eight hours of work regardless of the number of man-hours used. What is the dual price for man-hours? Interpret.
- On the other hand, aluminum and copper are resources that are ordered as needed. The unit profit coefficients were determined by: (selling price per unit) - (cost of the resources per unit). The 10 units of aluminum cost the company \$100. What is the most the company should be willing to pay for extra aluminum?

ANS:

- 6 product 1, 4 product 2, Profit = \$540
- Between \$50 and \$75; at \$70 the profit is \$580
- No; total % change is $83 \frac{1}{3}\% < 100\%$
- Dual prices are the shadow prices for the resources; since there was unused copper (because $S_2 = 2$), extra copper is worth \$0
- \$30
- \$10; this is the amount extra man-hours are worth
- The shadow price is the "premium" for aluminum -- would be willing to pay up to $\$10 + \$30 = \$40$ for extra aluminum

PTS: 1

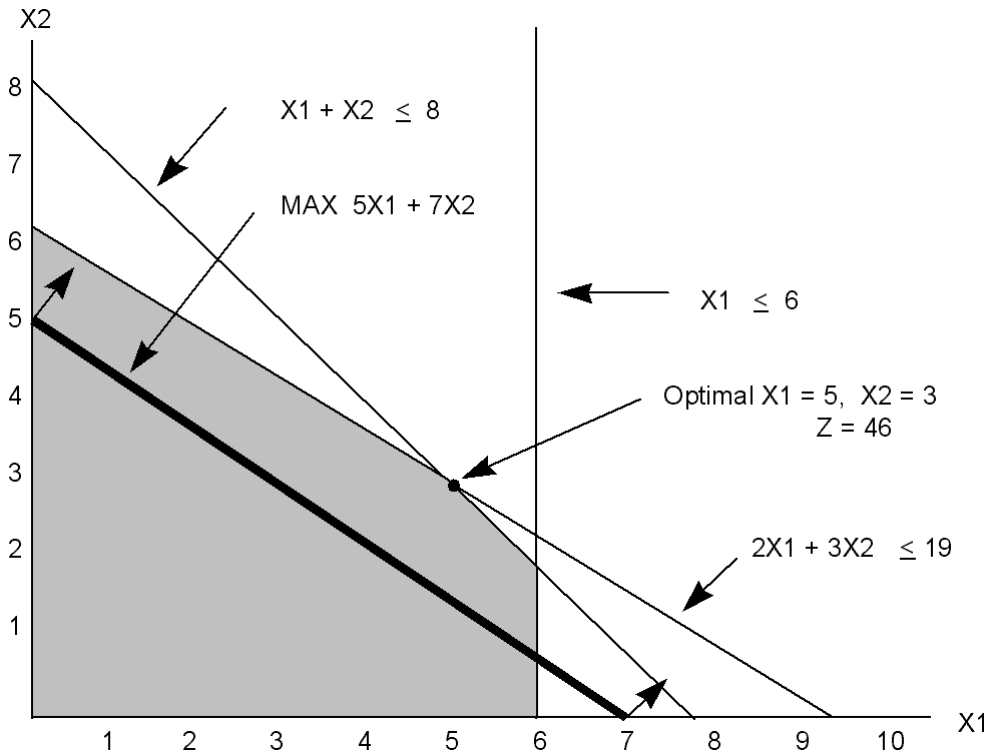
TOP: Interpretation of solution

18. Given the following linear program:

$$\text{Max } 5x_1 + 7x_2$$

$$\begin{aligned} \text{s.t. } & x_1 \leq 6 \\ & 2x_1 + 3x_2 \leq 19 \\ & x_1 + x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

The graphical solution to the problem is shown below. From the graph we see that the optimal solution occurs at $x_1 = 5$, $x_2 = 3$, and obj. func. = 46.



- Calculate the range of optimality for each objective function coefficient.
- Calculate the dual price for each resource.

ANS:

- Ranges of optimality: $14/3 \leq c_1 \leq 7$ and $5 \leq c_2 \leq 15/2$
- Summarizing, the dual price for the first resource is 0, for the second resource is 2, and for the third is 1

PTS: 1

TOP: Introduction to sensitivity analysis

19. Consider the following linear program:

$$\text{Max } 3x_1 + 4x_2 \text{ (\$ Profit)}$$

$$\text{s.t. } x_1 + 3x_2 \leq 12$$

$$2x_1 + x_2 \leq 8$$

$$x_1 \leq 3$$

$$x_1, x_2 \geq 0$$

The Management Scientist provided the following solution output:

OPTIMAL SOLUTION

Objective Function Value = 20.000

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X1	2.400	0.000
X2	3.200	0.000

<u>Constraint</u>	<u>Slack/Surplus</u>	<u>Dual Price</u>
1	0.000	1.000
2	0.000	1.000
3	0.600	0.000

OBJECTIVE COEFFICIENT RANGES

<u>Variable</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
X1	1.333	3.000	8.000
X2	1.500	4.000	9.000

RIGHT HAND SIDE RANGES

<u>Constraint</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
1	9.000	12.000	24.000
2	4.000	8.000	9.000
3	2.400	3.000	No Upper Limit

- What is the optimal solution including the optimal value of the objective function?
- Suppose the profit on x_1 is increased to \$7. Is the above solution still optimal? What is the value of the objective function when this unit profit is increased to \$7?
- If the unit profit on x_2 was \$10 instead of \$4, would the optimal solution change?
- If simultaneously the profit on x_1 was raised to \$5.5 and the profit on x_2 was reduced to \$3, would the current solution still remain optimal?

ANS:

- $x_1 = 2.4$ and $x_2 = 3.2$, and obj. func. = \$20.00.
- Optimal solution will not change. Optimal profit will equal \$29.60.
- Because 10 is outside the range of 1.5 to 9.0, the optimal solution likely would change.
- Sum of the change percentages is $50\% + 40\% = 90\%$. Since this does not exceed 100% the optimal solution would not change.

PTS: 1 TOP: Interpretation of solution

20. Consider the following linear program:

Min $6x_1 + 9x_2$ (\$ cost)

$$\begin{aligned}
 \text{s.t.} \quad & x_1 + 2x_2 \leq 8 \\
 & 10x_1 + 7.5x_2 \geq 30 \\
 & \quad \quad \quad x_2 \geq 2 \\
 & \quad \quad \quad x_1, x_2 \geq 0
 \end{aligned}$$

The Management Scientist provided the following solution output:

OPTIMAL SOLUTION

Objective Function Value = 27.000

<u>Variable</u>	<u>Value</u>	<u>Reduced Cost</u>
X1	1.500	0.000
X2	2.000	0.000

<u>Constraint</u>	<u>Slack/Surplus</u>	<u>Dual Price</u>
1	2.500	0.000
2	0.000	-0.600
3	0.000	-4.500

OBJECTIVE COEFFICIENT RANGES

<u>Variable</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
X1	0.000	6.000	12.000
X2	4.500	9.000	No Upper Limit

RIGHT HAND SIDE RANGES

<u>Constraint</u>	<u>Lower Limit</u>	<u>Current Value</u>	<u>Upper Limit</u>
1	5.500	8.000	No Upper Limit
2	15.000	30.000	55.000
3	0.000	2.000	4.000

- What is the optimal solution including the optimal value of the objective function?
- Suppose the unit cost of x_1 is decreased to \$4. Is the above solution still optimal? What is the value of the objective function when this unit cost is decreased to \$4?
- How much can the unit cost of x_2 be decreased without concern for the optimal solution changing?
- If simultaneously the cost of x_1 was raised to \$7.5 and the cost of x_2 was reduced to \$6, would the current solution still remain optimal?
- If the right-hand side of constraint 3 is increased by 1, what will be the effect on the optimal solution?

ANS:

- $x_1 = 1.5$ and $x_2 = 2.0$, and the objective function value = 27.00.
- 4 is within this range of 0 to 12, so the optimal solution will not change. Optimal total cost will be \$24.00.
- x_2 can fall to 4.5 without concern for the optimal solution changing.
- Sum of the change percentages is 91.7%. This does not exceed 100%, so the optimal solution would not change.
- The right-hand side remains within the range of feasibility, so there is no change in the optimal solution. However, the objective function value increases by \$4.50.

PTS: 1

TOP: Interpretation of solution