Chapter 10—Distribution and Network Models

MULTIPLE CHOICE

1. The problem which deals with the distribution of goods from several sources to several destinations is the
   a. maximal flow problem
   b. transportation problem
   c. assignment problem
   d. shortest-route problem

   ANS: B PTS: 1 TOP: Transportation problem

2. The parts of a network that represent the origins are
   a. the capacities
   b. the flows
   c. the nodes
   d. the arcs

   ANS: C PTS: 1 TOP: Transportation problem

3. The objective of the transportation problem is to
   a. identify one origin that can satisfy total demand at the destinations and at the same time minimize total shipping cost.
   b. minimize the number of origins used to satisfy total demand at the destinations.
   c. minimize the number of shipments necessary to satisfy total demand at the destinations.
   d. minimize the cost of shipping products from several origins to several destinations.

   ANS: D PTS: 1 TOP: Transportation problem

4. The number of units shipped from origin *i* to destination *j* is represented by
   a. x_{ij}
   b. x_{ji}
   c. c_{ij}
   d. c_{ji}

   ANS: A PTS: 1 TOP: Transportation problem

5. Which of the following is not true regarding the linear programming formulation of a transportation problem?
   a. Costs appear only in the objective function.
   b. The number of variables is (number of origins) x (number of destinations).
   c. The number of constraints is (number of origins) x (number of destinations).
   d. The constraints' left-hand side coefficients are either 0 or 1.

   ANS: C PTS: 1 TOP: Transportation problem

6. The difference between the transportation and assignment problems is that
   a. total supply must equal total demand in the transportation problem
   b. the number of origins must equal the number of destinations in the transportation problem
   c. each supply and demand value is 1 in the assignment problem
   d. there are many differences between the transportation and assignment problems

   ANS: C PTS: 1 TOP: Assignment problem

7. In the general linear programming model of the assignment problem,
   a. one agent can do parts of several tasks.
   b. one task can be done by several agents.
   c. each agent is assigned to its own best task.
   d. one agent is assigned to one and only one task.

   ANS: D PTS: 1 TOP: Assignment problem

8. The assignment problem is a special case of the
   a. transportation problem.
   b. transshipment problem.
   c. maximal flow problem.
   d. shortest-route problem.

   ANS: A PTS: 1 TOP: Assignment problem

9. Which of the following is not true regarding an LP model of the assignment problem?
   a. Costs appear in the objective function only.
   b. All constraints are of the ≥ form.
   c. All constraint left-hand side coefficient values are 1.
   d. All decision variable values are either 0 or 1.

   ANS: B PTS: 1 TOP: Assignment problem

10. The assignment problem constraint $x_{31} + x_{32} + x_{33} + x_{34} \leq 2$ means
    a. agent 3 can be assigned to 2 tasks.
    b. agent 2 can be assigned to 3 tasks.
    c. a mixture of agents 1, 2, 3, and 4 will be assigned to tasks.
    d. there is no feasible solution.

    ANS: A PTS: 1 TOP: Assignment problem

11. Arcs in a transshipment problem
    a. must connect every node to a transshipment node.
    b. represent the cost of shipments.
    c. indicate the direction of the flow.
    d. All of the alternatives are correct.

    ANS: C PTS: 1 TOP: Transshipment problem

12. Constraints in a transshipment problem
    a. correspond to arcs.
    b. include a variable for every arc.
    c. require the sum of the shipments out of an origin node to equal supply.
    d. All of the alternatives are correct.

    ANS: B PTS: 1 TOP: Transshipment problem

13. In a transshipment problem, shipments
    a. cannot occur between two origin nodes.
    b. cannot occur between an origin node and a destination node.
    c. cannot occur between a transshipment node and a destination node.
    d. can occur between any two nodes.

    ANS: D PTS: 1 TOP: Transshipment problem

14. Consider a shortest route problem in which a bank courier must travel between branches and the main operations center. When represented with a network,
    a. the branches are the arcs and the operations center is the node.
    b. the branches are the nodes and the operations center is the source.
    c. the branches and the operations center are all nodes and the streets are the arcs.
    d. the branches are the network and the operations center is the node.

    ANS: C PTS: 1 TOP: Shortest-route problem

15. The shortest-route problem finds the shortest-route
    a. from the source to the sink.
    b. from the source to any other node.
    c. from any node to any other node.
    d. from any node to the sink.

    ANS: B PTS: 1 TOP: Shortest-route problem

16. Consider a maximal flow problem in which vehicle traffic entering a city is routed among several routes before eventually leaving the city. When represented with a network,
    a. the nodes represent stoplights.
    b. the arcs represent one way streets.
    c. the nodes represent locations where speed limits change.
    d. None of the alternatives is correct.

    ANS: B PTS: 1 TOP: Maximal flow problem

17. We assume in the maximal flow problem that
    a. the flow out of a node is equal to the flow into the node.
    b. the source and sink nodes are at opposite ends of the network.
    c. the number of arcs entering a node is equal to the number of arcs exiting the node.
    d. None of the alternatives is correct.

    ANS: A PTS: 1 TOP: Maximal flow problem

18. If a transportation problem has four origins and five destinations, the LP formulation of the problem will have
    a. 5 constraints
    b. 9 constraints
    c. 18 constraints
    d. 20 constraints

    ANS: B PTS: 1 TOP: Transportation problem

19. Which of the following is NOT a characteristic of assignment problems?
    a. costs appear in the objective function only
    b. the RHS of constraints are all 1
    c. the value of all decision variables is either 0 or 1
    d. the signs of constraints are always $\leq$

    ANS: D PTS: 1 TOP: Assignment problem

20. The network flows into and out of demand nodes are what makes the production and inventory application modeled in the textbook a
    a. shortest-route model
    b. maximal flow model

c. transportation model
d. transshipment model

ANS: D PTS: 1 TOP: A production and inventory application

TRUE/FALSE

1. Whenever total supply is less than total demand in a transportation problem, the LP model does not determine how the unsatisfied demand is handled.

   ANS: T PTS: 1 TOP: Transportation problem

2. Converting a transportation problem LP from cost minimization to profit maximization requires only changing the objective function; the conversion does not affect the constraints.

   ANS: T PTS: 1 TOP: Transportation problem

3. A transportation problem with 3 sources and 4 destinations will have 7 decision variables.

   ANS: F PTS: 1 TOP: Transportation problem

4. If a transportation problem has four origins and five destinations, the LP formulation of the problem will have nine constraints.

   ANS: T PTS: 1 TOP: Transportation problem

5. The capacitated transportation problem includes constraints which reflect limited capacity on a route.

   ANS: T PTS: 1 TOP: Transportation problem

6. When the number of agents exceeds the number of tasks in an assignment problem, one or more dummy tasks must be introduced in the LP formulation or else the LP will not have a feasible solution.

   ANS: F PTS: 1 TOP: Assignment problem

7. A transshipment constraint must contain a variable for every arc entering or leaving the node.

   ANS: T PTS: 1 TOP: Transshipment problem

8. The shortest-route problem is a special case of the transshipment problem.

   ANS: T PTS: 1 TOP: Shortest-route problem

9. Transshipment problem allows shipments both in and out of some nodes while transportation problems do not.

   ANS: T PTS: 1 TOP: Transportation and transshipment problems

10. A dummy origin in a transportation problem is used when supply exceeds demand.

    ANS: F PTS: 1 TOP: Transportation problem

11. When a route in a transportation problem is unacceptable, the corresponding variable can be removed from the LP formulation.

    ANS: T PTS: 1 TOP: Transportation problem

12. In the LP formulation of a maximal flow problem, a conservation-of-flow constraint ensures that an arc's flow capacity is not exceeded.

    ANS: F PTS: 1 TOP: Maximal flow problem

13. The maximal flow problem can be formulated as a capacitated transshipment problem.

    ANS: T PTS: 1 TOP: Maximal flow problem

14. The direction of flow in the shortest-route problem is always out of the origin node and into the destination node.

    ANS: T PTS: 1 TOP: Shortest-route problem

15. A transshipment problem is a generalization of the transportation problem in which certain nodes are neither supply nodes nor destination nodes.

    ANS: T PTS: 1 TOP: Transshipment problem

16. The assignment problem is a special case of the transportation problem in which all supply and demand values equal one.

    ANS: T PTS: 1 TOP: Assignment problem

17. A transportation problem with 3 sources and 4 destinations will have 7 variables in the objective function.

    ANS: F PTS: 1 TOP: Assignment problem

18. Flow in a transportation network is limited to one direction.

    ANS: T PTS: 1 TOP: Transportation problem

19. In a transportation problem with total supply equal to total demand, if there are four origins and seven destinations, and there is a unique optimal solution, the optimal solution will utilize 11 shipping routes.

    ANS: F PTS: 1 TOP: Transportation problem

20. In the general assignment problem, one agent can be assigned to several tasks.

    ANS: T PTS: 1 TOP: Assignment problem

SHORT ANSWER

1. Explain how the general linear programming model of the assignment problem can be modified to handle problems involving a maximization function, unacceptable assignments, and supply not equally demand.

ANS:
Answer not provided.

PTS: 1 TOP: Assignment problem

2. Define the variables and constraints necessary in the LP formulation of the transshipment problem.

ANS:
Answer not provided.

PTS: 1 TOP: Transshipment problem

3. Explain what adjustments are made to the transportation linear program when there are unacceptable routes.

ANS:
Answer not provided.

PTS: 1 TOP: Transportation problem

4. Is it a coincidence to obtain integer solutions to network problems? Explain.

ANS:
Answer not provided.

PTS: 1 TOP: Network problems

5. How is the assignment linear program different from the transportation model?

ANS:
Answer not provided.

PTS: 1 TOP: Transportation and assignment problems

6. Define the variables and constraints necessary in the LP formulation of the maximal flow problem.

ANS:
Answer not provided.

PTS: 1 TOP: Maximal flow problem

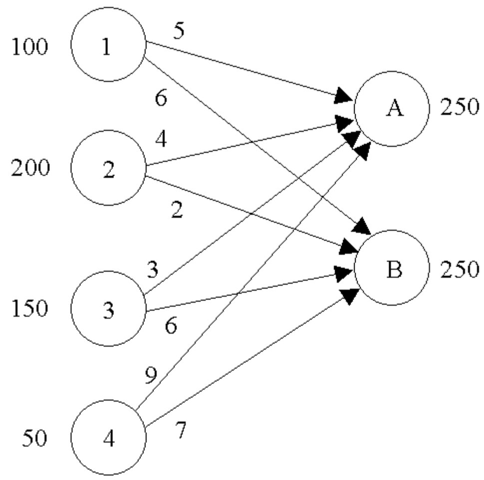
7. How is the shortest-route problem like the transshipment problem?

ANS:
Answer not provided.

PTS: 1 TOP: Shortest-route problem

PROBLEM

1. Write the LP formulation for this transportation problem.



ANS:

Min $5x_{1A} + 6x_{1B} + 4x_{2A} + 2x_{2B} + 3x_{3A} + 6x_{3B} + 9x_{4A} + 7x_{4B}$

s.t.
$x_{1A} + x_{1B} \le 100$
$x_{2A} + x_{2B} \le 200$
$x_{3A} + x_{3B} \le 150$
$x_{4A} + x_{4B} \le 50$
$x_{1A} + x_{2A} + x_{3A} + x_{4A} = 250$
$x_{1B} + x_{2B} + x_{3B} + x_{4B} = 250$
all $x_{ij} \ge 0$

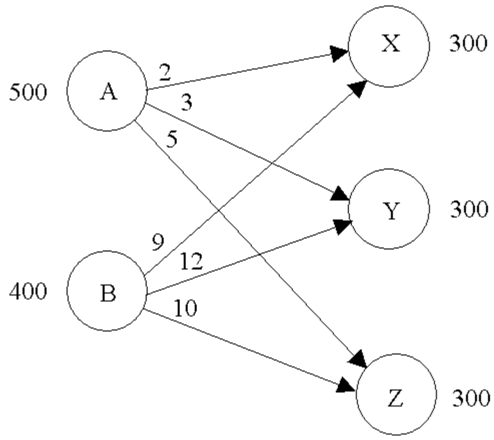
PTS: 1 TOP: Transportation problem

2. Draw the network for this transportation problem.

Min $2x_{AX} + 3x_{AY} + 5x_{AZ} + 9x_{BX} + 12x_{BY} + 10x_{BZ}$

s.t.
$x_{AX} + x_{AY} + x_{AZ} \le 500$
$x_{BX} + x_{BY} + x_{BZ} \le 400$
$x_{AX} + x_{BX} = 300$
$x_{AY} + x_{BY} = 300$
$x_{AZ} + x_{BZ} = 300$
$x_{ij} \ge 0$

ANS:



PTS: 1 TOP: Transportation problem

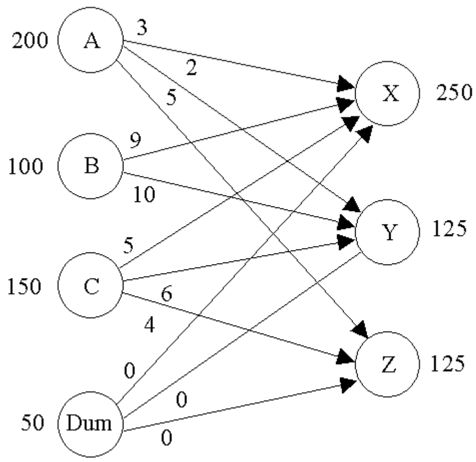
3. Canning Transport is to move goods from three factories to three distribution centers. Information about the move is given below. Give the network model and the linear programming model for this problem.

Source	*Supply*	*Destination*	*Demand*
A	200	X	50
B	100	Y	125
C	150	Z	125

Shipping costs are:

	Destination		
Source	X	Y	Z
A	3	2	5
B	9	10	--
C	5	6	4
	(Source B cannot ship to destination Z)		

ANS:



Min $3x_{AX} + 2x_{AY} + 5x_{AZ} + 9x_{BX} + 10x_{BY} + 5x_{CX} + 6x_{CY} + 4x_{CZ}$

s.t. $x_{AX} + x_{AY} + x_{AZ} \leq 200$

$x_{BX} + x_{BY} \leq 100$

$$x_{CX} + x_{CY} + x_{CZ} \leq 150$$
$$x_{DX} + x_{DY} + x_{DZ} \leq 50$$
$$x_{AX} + x_{BX} + x_{CX} + x_{DX} = 250$$
$$x_{AY} + x_{BY} + x_{CY} + x_{DY} = 125$$
$$x_{AZ} + x_{BZ} + x_{CZ} + x_{DZ} = 125$$
$$x_{ij} \geq 0$$

PTS: 1 TOP: Transportation problem

4. The following table shows the unit shipping cost between cities, the supply at each source city, and the demand at each destination city. The Management Scientist solution is shown. Report the optimal solution.

	Destination				
Source	Terre Haute	Indianapolis	Ft. Wayne	South Bend	*Supply*
St. Louis	8	6	12	9	100
Evansville	5	5	10	8	100
Bloomington	3	2	9	10	100
Demand	150	60	45	45	

TRANSPORTATION PROBLEM
****

OBJECTIVE: MINIMIZATION

SUMMARY OF ORIGIN SUPPLIES
****

ORIGIN	SUPPLY
1	100
2	100
3	100

SUMMARY OF DESTINATION DEMANDS
****

DESTINATION	DEMAND
1	150
2	60
3	45
4	45

SUMMARY OF UNIT COST OR REVENUE DATA
****

FROM	TO DESTINATION			
ORIGIN	1	2	3	4
1	8	6	12	9
2	5	5	10	8
3	3	2	9	10

OPTIMAL TRANSPORTATION SCHEDULE
****

SHIP
FROM TO DESTINATION

ORIGIN	1	2	3	4
1	0	10	45	45
2	100	0	0	0
3	50	50	0	0

TOTAL TRANSPORTATION COST OR REVENUE IS 1755

ANS:
Ship 10 from St. Louis to Indianapolis, 45 from St. Louis to Ft. Wayne, 45 from St. Louis to South Bend, 100 from Evansville to Terre Haute, 50 from Bloomington to Terre Haute, and 50 from Bloomington to Indianapolis. The total cost is 1755.

PTS: 1 TOP: Transportation problem

5. After some special presentations, the employees of the AV Center have to move overhead projectors back to classrooms. The table below indicates the buildings where the projectors are now (the sources), where they need to go (the destinations), and a measure of the distance between sites.

	Destination				
Source	Business	Education	Parsons Hall	Holmstedt Hall	*Supply*
Baker Hall	10	9	5	2	35
Tirey Hall	12	11	1	6	10
Arena	15	14	7	6	20
Demand	12	20	10	10	

a. If you were going to write this as a linear programming model, how many decision variables would there be, and how many constraints would there be?

The solution to this problem is shown below. Use it to answer the questions b - e.

TRANSPORTATION PROBLEM
****
OPTIMAL TRANSPORTATION SCHEDULE
****
SHIP
FROM TO DESTINATION

ORIGIN	1	2	3	4
1	12	20	0	3
2	0	0	10	0
3	0	0	0	7

TOTAL TRANSPORTATION COST OR REVENUE IS 358
NOTE: THE TOTAL SUPPLY EXCEEDS THE TOTAL DEMAND BY 13

ORIGIN EXCESS SUPPLY

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3 13

b. How many projectors are moved from Baker to Business?
c. How many projectors are moved from Tirey to Parsons?
d. How many projectors are moved from the Arena to Education?
e. Which site(s) has (have) projectors left?

ANS:

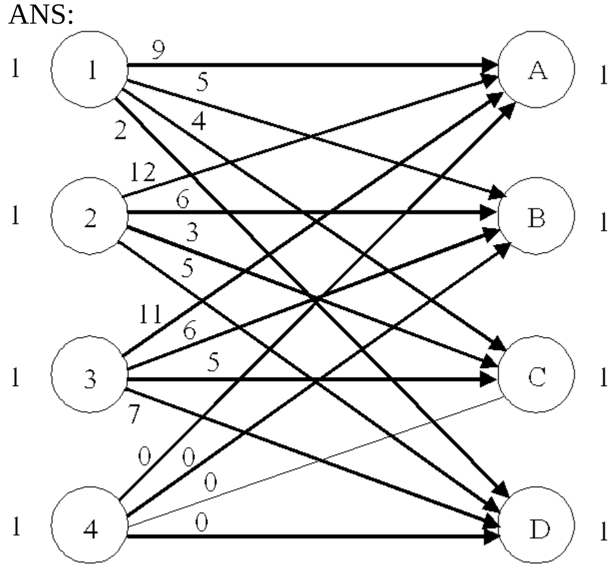
a. 12 decision variables, 7 constraints
b. 12
c. 10
d. 0
e. Arena

PTS: 1 TOP: Transportation problem

6. Show both the network and the linear programming formulation for this assignment problem.

	Task			
Person	A	B	C	D
1	9	5	4	2
2	12	6	3	5
3	11	6	5	7

ANS:



Let x_{ij} = 1 if person i is assigned to job j
= 0 otherwise

Min $9x_{1A} + 5x_{1B} + 4x_{1C} + 2x_{1D}$
$+ 12x_{2A} + 6x_{2B} + 3x_{2C} + 5x_{2D}$
$+ 11x_{3A} + 6x_{3B} + 5x_{3C} + 7x_{3D}$

s.t. $x_{1A} + x_{1B} + x_{1C} + x_{1D} \leq 1$
$x_{2A} + x_{2B} + x_{2C} + x_{2D} \leq 1$

$x_{3A} + x_{3B} + x_{3C} + x_{3D} \leq 1$
$x_{4A} + x_{4B} + x_{4C} + x_{4D} \leq 1$
$x_{1A} + x_{2A} + x_{3A} + x_{4A} = 1$
$x_{1B} + x_{2B} + x_{3B} + x_{4B} = 1$
$x_{1C} + x_{2C} + x_{3C} + x_{4C} = 1$
$x_{1D} + x_{2D} + x_{3D} + x_{4D} = 1$

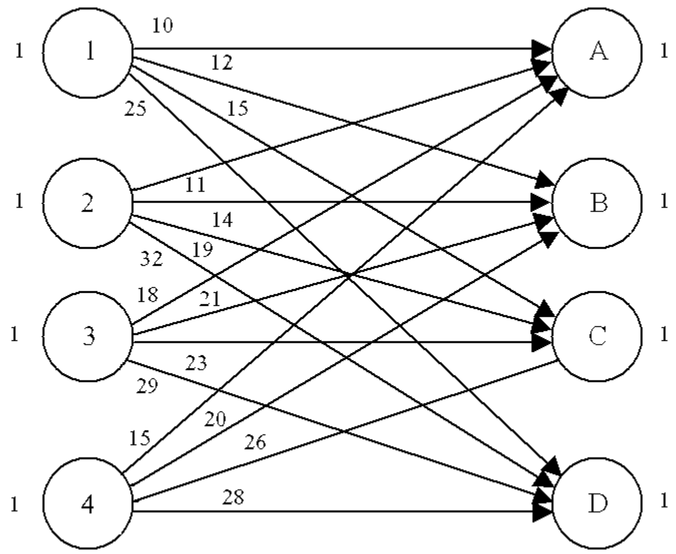
PTS: 1 TOP: Assignment problem

7. Draw the network for this assignment problem.

Min $10x_{1A} + 12x_{1B} + 15x_{1C} + 25x_{1D} + 11x_{2A} + 14x_{2B} + 19x_{2C} + 32x_{2D}$
$+ 18x_{3A} + 21x_{3B} + 23x_{3C} + 29x_{3D} + 15x_{4A} + 20x_{4B} + 26x_{4C} + 28x_{4D}$

s.t. $x_{1A} + x_{1B} + x_{1C} + x_{1D} = 1$
$x_{2A} + x_{2B} + x_{2C} + x_{2D} = 1$
$x_{3A} + x_{3B} + x_{3C} + x_{3D} = 1$
$x_{4A} + x_{4B} + x_{4C} + x_{4D} = 1$
$x_{1A} + x_{2A} + x_{3A} + x_{4A} = 1$
$x_{1B} + x_{2B} + x_{3B} + x_{4B} = 1$
$x_{1C} + x_{2C} + x_{3C} + x_{4C} = 1$
$x_{1D} + x_{2D} + x_{3D} + x_{4D} = 1$

ANS:



PTS: 1 TOP: Assignment problem

8. A professor has been contacted by four not-for-profit agencies that are willing to work with student consulting teams. The agencies need help with such things as budgeting, information systems, coordinating volunteers, and forecasting. Although each of the four student teams could work with any of the agencies, the professor feels that there is a difference in the amount of time it would take each group to solve each problem. The professor's estimate of the time, in days, is given in the table below. Use the computer solution to see which team works with which project.

	Projects			
Team	Budgeting	Information	Volunteers	Forecasting
A	32	35	15	27

B	38	40	18	35
C	41	42	25	38
D	45	45	30	42

ASSIGNMENT PROBLEM
****

OBJECTIVE: MINIMIZATION

SUMMARY OF UNIT COST OR REVENUE DATA
****

	TASK			
AGENT	1	2	3	4
----------	-----	-----	-----	-----
1	32	35	15	27
2	38	40	18	35
3	41	42	25	38
4	45	45	30	42

OPTIMAL ASSIGNMENTS COST/REVENUE
*********************** ***************

ASSIGN AGENT 3 TO TASK 1	41
ASSIGN AGENT 4 TO TASK 2	45
ASSIGN AGENT 2 TO TASK 3	18
ASSIGN AGENT 1 TO TASK 4	27
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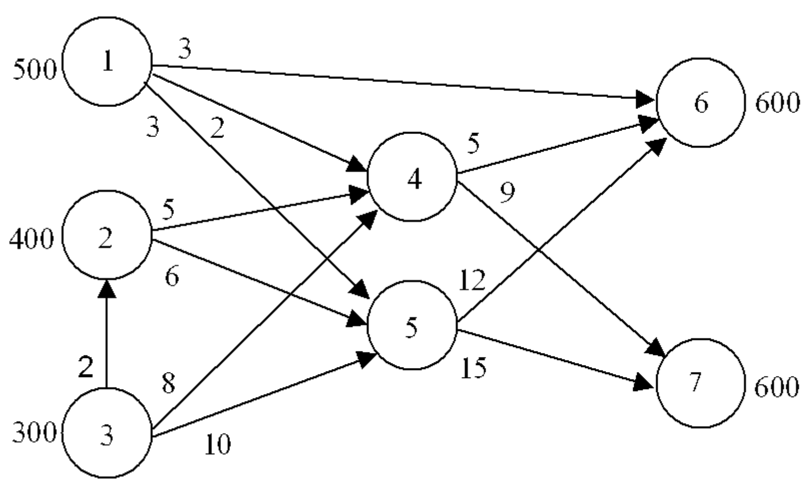
TOTAL COST/REVENUE	131

ANS:
Team A works with the forecast, Team B works with volunteers, Team C works with budgeting, and Team D works with information. The total time is 131.

PTS: 1 TOP: Assignment problem

9. Write the linear program for this transshipment problem.



ANS:

Min $3x_{16} + 2x_{14} + 3x_{15} + 5x_{24} + 6x_{25} + 2x_{32} + 8x_{34} + 10x_{35} + 5x_{46} + 9x_{47} + 12x_{56} + 15x_{57}$

s.t.
$x_{16} + x_{14} + x_{35} \leq 500$
$x_{24} + x_{25} - x_{23} \leq 400$
$x_{32} + x_{34} + x_{35} \leq 300$
$x_{46} + x_{47} - (x_{14} + x_{24} + x_{34}) = 0$
$x_{56} + x_{57} - (x_{15} + x_{25} + x_{35}) = 0$
$x_{16} + x_{46} + x_{56} = 600$
$x_{56} + x_{57} = 600$

PTS: 1 TOP: Transshipment problem

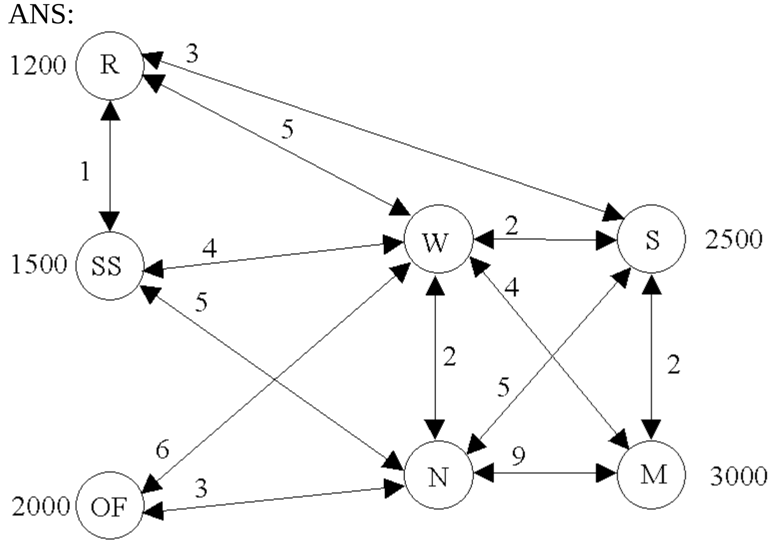
10. Peaches are to be transported from three orchard regions to two canneries. Intermediate stops at a consolidation station are possible.

Orchard	Supply	Station	Cannery	Capacity
Riverside	1200	Waterford	Sanderson	2500
Sunny Slope	1500	Northside	Millville	3000
Old Farm	2000			

Shipment costs are shown in the table below. Where no cost is given, shipments are not possible. Where costs are shown, shipments are possible in either direction. Draw the network model for this problem.

	R	SS	OF	W	N	S	M
Riverside		1		5		3	
Sunny Side				4	5		
Old Farm				6	3		
Waterford					2	2	4
Northside						5	9
Sanderson							2
Millville							

ANS:



PTS: 1 TOP: Transshipment problem

11. RVW (Restored Volkswagens) buys 15 used VW's at each of two car auctions each week held at different locations. It then transports the cars to repair shops it contracts with. When they are restored to RVW's specifications, RVW sells 10 each to three different used car lots. There are various costs associated with the average purchase and transportation prices from each auction to each repair shop. Also there are transportation costs from the repair shops to the used car lots. RVW is concerned with minimizing its total cost given the costs in the table below.

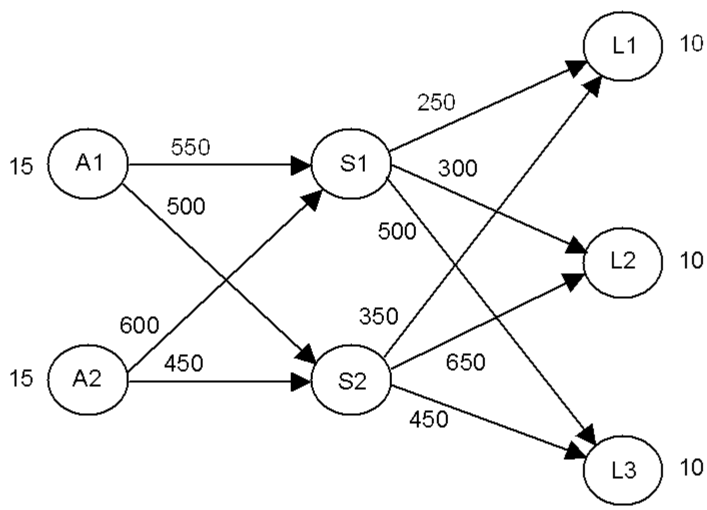
a. Given the costs below, draw a network representation for this problem.

	Repair Shops			Used Car Lots		
	S1	S2		L1	L2	L3
Auction 1	550	500	S1	250	300	500
Auction 2	600	450	S2	350	650	450

b. Formulate this problem as a transshipment linear programming model.

ANS:

a.



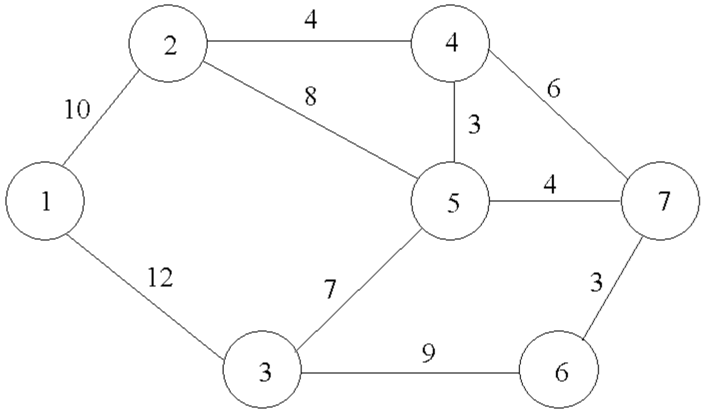
b. Denote A1 as node 1, A2 as node 2, S1 as node 3, S2 as node 4, L1 as node 5, L2 as node 6, and L3 as node 7

Min $50x_{13} + 500x_{14} + 600x_{23} + 450x_{24} + 250x_{35} + 300x_{36}$
$+ 500x_{37} + 350x_{45} + 650x_{46} + 450x_{47}$

s.t.
$x_{13} + x_{14} \leq 15$
$x_{23} + x_{24} \leq 15$
$x_{13} + x_{23} - x_{35} - x_{36} - x_{37} = 0$
$x_{14} + x_{24} - x_{45} - x_{46} - x_{47} = 0$
$x_{35} + x_{45} = 10$
$x_{36} + x_{46} = 10$
$x_{37} + x_{4} = 10$
$x_{ij} \geq 0$ for all i,j

PTS: 1 TOP: Transshipment problem

12. Consider the network below. Formulate the LP for finding the shortest-route path from node 1 to node 7.



ANS:

Min $10x_{12} + 12x_{13} + 4x_{24} + 8x_{25} + 7x_{35} + 9x_{36} + 4x_{42} + 3x_{45}$
$+ 6x_{47} + 8x_{52} + 7x_{53} + 3x_{54} + 4x_{57} + 9x_{63} + 3x_{67}$

s.t.

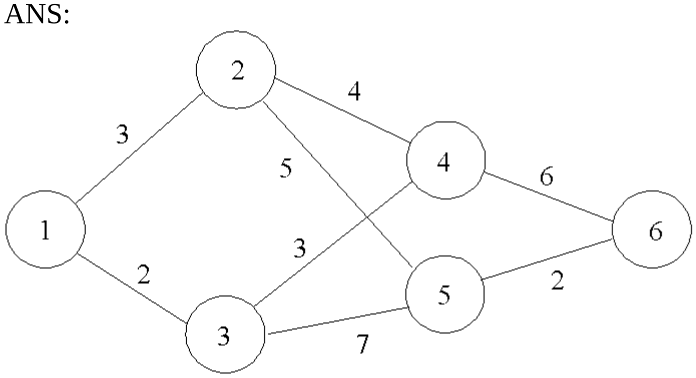
$$
\begin{aligned}
x_{12} + x_{13} &= 1 \\
-x_{12} + x_{24} + x_{25} - x_{42} - x_{52} &= 0 \\
-x_{13} + x_{35} + x_{36} - x_{53} - x_{63} &= 0 \\
-x_{24} + x_{42} + x_{45} + x_{47} - x_{54} &= 0 \\
-x_{25} - x_{35} - x_{45} + x_{52} + x_{53} + x_{57} &= 0 \\
-x_{36} + x_{63} + x_{67} &= 0 \\
x_{47} + x_{57} + x_{67} &= 1 \\
x_{ij} \geq 0 \text{ for all } i,j &
\end{aligned}
$$

PTS: 1 TOP: Shortest-route problem

13. Consider the following shortest-route problem involving six cities with the distances given. Draw the network for this problem and formulate the LP for finding the shortest distance from City 1 to City 6.

Path	Distance
1 to 2	3
1 to 3	2
2 to 4	4
2 to 5	5
3 to 4	3
3 to 5	7
4 to 6	6
5 to 6	2

ANS:

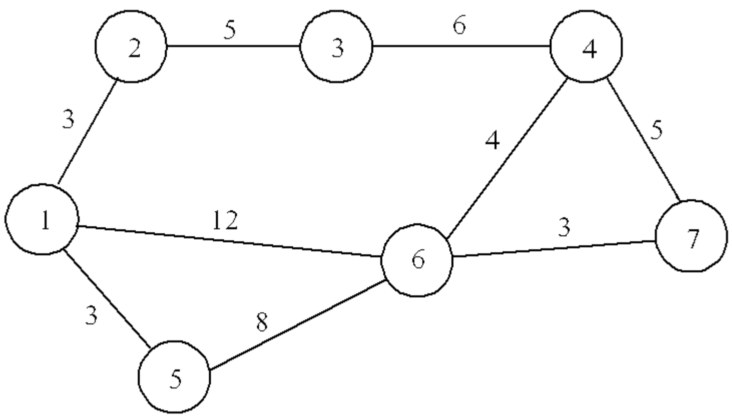


Min $3x_{12} + 2x_{13} + 4x_{24} + 5x_{25} + 3x_{34} + 7x_{35}$
$+ 4x_{42} + 3x_{43} + 6x_{46} + 5x_{52} + 7x_{53} + 2x_{56}$

s.t.
$x_{12} + x_{13} = 1$
$-x_{12} + x_{24} + x_{25} - x_{42} - x_{52} = 0$
$-x_{13} + x_{34} + x_{35} - x_{43} - x_{53} = 0$
$-x_{24} - x_{34} + x_{42} + x_{43} + x_{46} = 0$
$-x_{25} - x_{35} + x_{52} + x_{53} + x_{56} = 0$
$x_{46} + x_{56} = 1$
$x_{ij} \geq 0$ for all i,j

PTS: 1 TOP: Shortest-route problem

14. A beer distributor needs to plan how to make deliveries from its warehouse (Node 1) to a supermarket (Node 7), as shown in the network below. Develop the LP formulation for finding the shortest route from the warehouse to the supermarket.



ANS:

Min $3x_{12} + 3x_{15} + 12x_{16} + 5x_{23} + 5x_{32} + 6x_{34} + 6x_{43}$
$+ 4x_{46} + 5x_{47} + 8x_{56} + 4x_{64} + 8x_{65} + 3x_{67}$

s.t.
$x_{12} + x_{15} + x_{16} = 1$
$-x_{12} + x_{23} = 0$
$-x_{23} + x_{32} + x_{34} = 0$
$-x_{34} + x_{43} + x_{46} + x_{47} - 4x_{64} = 0$
$-x_{15} + x_{56} = 0$
$-x_{16} - x_{46} - x_{56} + x_{64} + x_{65} + x_{67} = 0$
$x_{47} + x_{67} = 1$
$x_{ij} \geq 0$ for all i,j

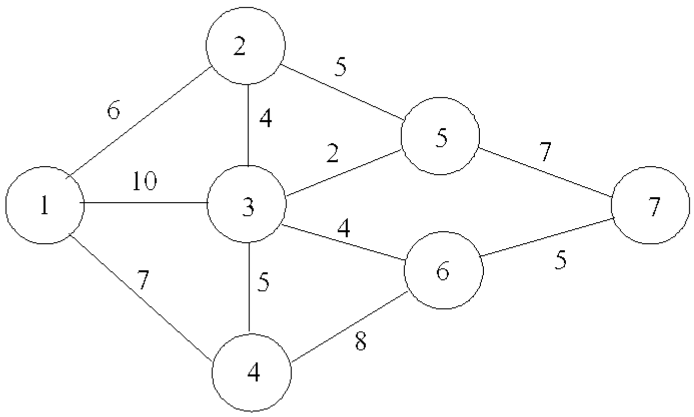
PTS: 1 TOP: Shortest-route problem

15. Consider the following shortest-route problem involving seven cities. The distances between the cities are given below. Draw the network model for this problem and formulate the LP for finding the shortest route from City 1 to City 7.

Path	Distance
1 to 2	6

1 to 3	10
1 to 4	7
2 to 3	4
2 to 5	5
3 to 4	5
3 to 5	2
3 to 6	4
4 to 6	8
5 to 7	7
6 to 7	5

ANS:



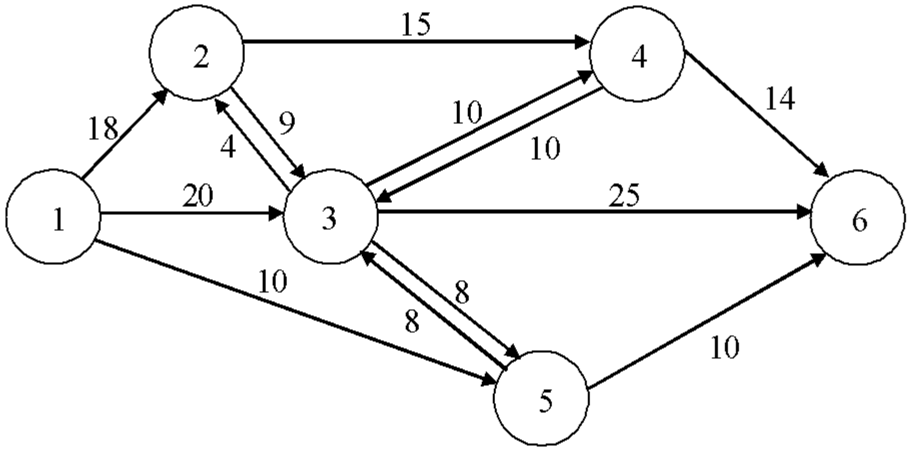
Min $6x_{12} + 10x_{13} + 7x_{14} + 4x_{23} + 5x_{25} + 4x_{32} + 5x_{34} + 2x_{35} + 4x_{36} + 5x_{43} + 8x_{46} + 5x_{52} + 2x_{53} + 7x_{57} + 4x_{63} + 8x_{64} + 5x_{67}$

s.t.

$$x_{12} + x_{13} + x_{14} = 1$$
$$-x_{12} + x_{23} + x_{25} - x_{32} - x_{52} = 0$$
$$-x_{13} - x_{23} + x_{32} + x_{34} + x_{35} + x_{36} - x_{43} - x_{53} - x_{63} = 0$$
$$-x_{14} - x_{34} + x_{43} + x_{46} - x_{64} = 0$$
$$-x_{25} - x_{35} + x_{52} + x_{53} + x_{57} = 0$$
$$-x_{36} - x_{46} + x_{63} + x_{64} + x_{67} = 0$$
$$x_{57} + x_{67} = 1$$
$$x_{ij} \geq 0 \text{ for all } i,j$$

PTS: 1 TOP: Shortest-route problem

16. The network below shows the flows possible between pairs of six locations. Formulate an LP to find the maximal flow possible from Node 1 to Node 6.



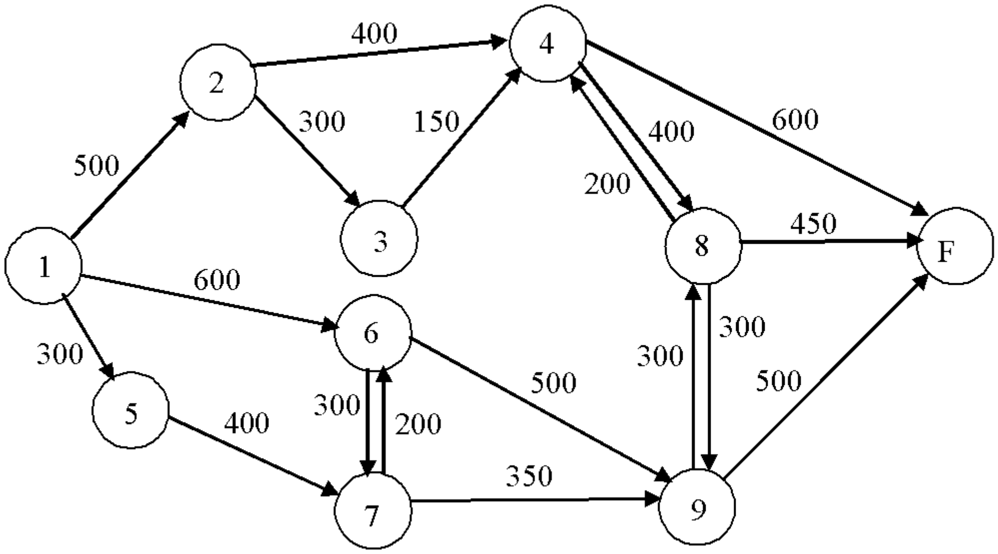
ANS:

Min x_{61}

s.t.
$x_{12} + x_{13} + x_{15} - x_{61} = 0$
$x_{23} + x_{24} - x_{12} - x_{32} = 0$
$x_{32} + x_{34} + x_{35} - x_{13} - x_{23} - x_{53} = 0$
$x_{43} + x_{46} - x_{24} - x_{34} = 0$
$x_{53} + x_{56} - x_{15} - x_{35} = 0$
$x_{61} - x_{36} - x_{46} - x_{56} = 0$
$x_{12} \leq 18$ $x_{13} \leq 20$ $x_{15} \leq 10$
$x_{23} \leq 9$ $x_{24} \leq 15$
$x_{32} \leq 4$ $x_{34} \leq 10$ $x_{35} \leq 8$
$x_{43} \leq 10$ $x_{46} \leq 14$
$x_{53} \leq 8$ $x_{56} \leq 10$
$x_{ij} \geq 0$ for all i,j

PTS: 1 TOP: Maximal flow problem

17. A network of railway lines connects the main lines entering and leaving a city. Speed limits, track reconstruction, and train length restrictions lead to the flow diagram below, where the numbers represent how many cars can pass per hour. Formulate an LP to find the maximal flow in cars per hour from Node 1 to Node F.



ANS:

Min x_{F1}

s.t.

$x_{12} + x_{15} + x_{16} - x_{F1} = 0$
$x_{23} + x_{24} - x_{12} = 0$
$x_{34} - x_{23} = 0$
$x_{48} + x_{4F} - x_{24} - x_{34} - x_{84} = 0$
$x_{57} - x_{15} = 0$
$x_{67} + x_{69} - x_{16} - x_{76} = 0$
$x_{76} + x_{79} - x_{57} - x_{67} = 0$
$x_{84} + x_{89} + x_{8F} - x_{48} - x_{98} = 0$
$x_{98} + x_{9F} - x_{69} - x_{79} - x_{89} = 0$
$x_{F1} - x_{4F} - x_{8F} - x_{9F} = 0$

$x_{12} \le 500$	$x_{15} \le 300$	$x_{16} \le 600$
$x_{23} \le 300$	$x_{24} \le 400$	
$x_{34} \le 150$		
$x_{48} \le 400$	$x_{4F} \le 600$	
$x_{57} \le 400$		
$x_{67} \le 300$	$x_{69} \le 500$	
$x_{76} \le 200$	$x_{79} \le 350$	
$x_{84} \le 200$	$x_{89} \le 300$	$x_{8F} \le 450$
$x_{98} \le 300$	$x_{9F} \le 500$	

$x_{ij} \ge 0$ for all i,j

PTS: 1 TOP: Maximal flow problem

18. Fodak must schedule its production of camera film for the first four months of the year. Film demand (in 1,000s of rolls) in January, February, March and April is expected to be 300, 500, 650 and 400, respectively. Fodak's production capacity is 500 thousand rolls of film per month. The film business is highly competitive, so Fodak cannot afford to lose sales or keep its customers waiting. Meeting month *i* 's demand with month *i* +1's production is unacceptable.

Film produced in month *i* can be used to meet demand in month i or can be held in inventory to meet demand in month *i* +1 or month *i* +2 (but not later due to the film's limited shelflife). There is no film in inventory at the start of January.

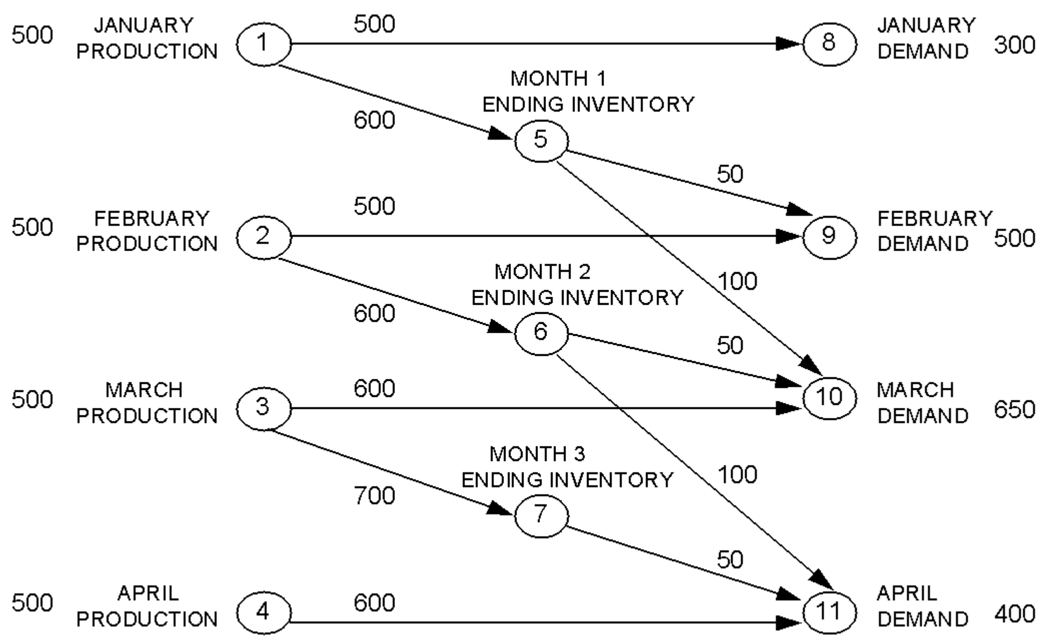
The film's production and delivery cost per thousand rolls will be $500 in January and February. This cost will increase to $600 in March and April due to a new labor contract. Any film put in inventory requires additional transport costing $100 per thousand rolls. It costs $50 per thousand rolls to hold film in inventory from one month to the next.

a. Modeling this problem as a transshipment problem, draw the network representation.

b. Formulate and solve this problem as a linear program.

ANS:

a.



b. <u>Define the decision variables</u>:

x_{ij} = amount of film "moving" between node *i* and node *j*

<u>Define objective</u>:

Min $600x_{15} + 500x_{18} + 600x_{26} + 500x_{29} + 700x_{37} + 600x_{310} + 600x_{411} + 50x_{59}$
$+ 100x_{510} + 50x_{610} + 100x_{611} + 50x_{711}$

<u>Define the constraints</u>:

Amount (1000's of rolls) of film produced in January:	$x_{15} + x_{18} \le 500$
Amount (1000's of rolls) of film produced in February:	$x_{26} + x_{29} \le 500$
Amount (1000's of rolls) of film produced in March:	$x_{37} + x_{310} \le 500$
Amount (1000's of rolls) of film produced in April:	$x_{411} \le 500$
Amount (1000's of rolls) of film in/out of January inventory:	$x_{15} - x_{59} - x_{510} = 0$
Amount (1000's of rolls) of film in/out of February inventory:	$x_{26} - x_{610} - x_{611} = 0$
Amount (1000's of rolls) of film in/out of March inventory:	$x_{37} - x_{711} = 0$
Amount (1000's of rolls) of film satisfying January demand:	$x_{18} = 300$
Amount (1000's of rolls) of film satisfying February demand:	$x_{29} + x_{59} = 500$
Amount (1000's of rolls) of film satisfying March demand:	$x_{310} + x_{510} + x_{610} = 650$
Amount (1000's of rolls) of film satisfying April demand:	$x_{411} + x_{611} + x_{711} = 400$

Non-negativity of variables: $x_{ij} \ge 0$, for all *i* and *j*.

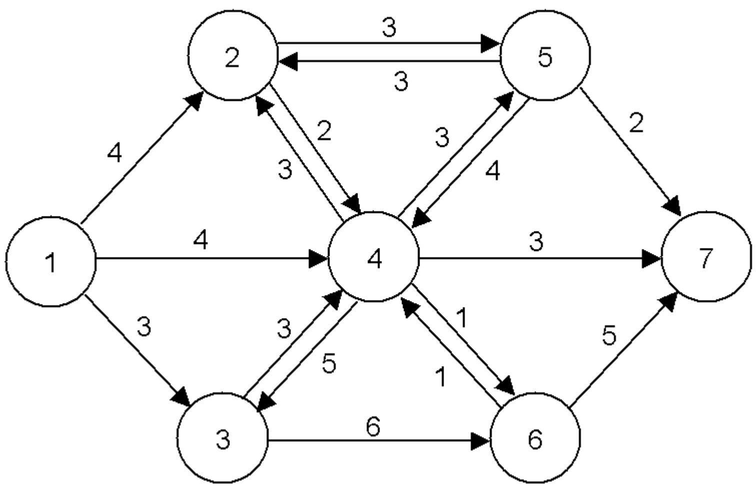
The Management Scientist provided the following solution:

Objective Function Value = 1045000.000

VARIABLE	VALUE	REDUCED COST
X15	150.000	0.000
X18	300.000	0.000
X26	0.000	100.000
X29	500.000	0.000
X37	0.000	250.000
X310	500.000	0.000
X411	400.000	0.000
X59	0.000	0.000
X510	150.000	0.000
X610	0.000	0.000
X611	0.000	150.000
X711	0.000	0.000

PTS: 1 TOP: Production and inventory application

19. Find the maximal flow from node 1 to node 7 in the following network.



ANS:
Decision variables:
x_{ij} = amount of flow from node *i* to node *j*
Objective function:
Maximize the flow through the network: Max x_{71}
Constraints:
Conservation of flow for each node:
(1) $x_{12} + x_{13} + x_{14} - x_{71} = 0$ (node 1)
(2) $x_{24} + x_{25} - x_{12} - x_{42} - x_{52} = 0$ (node 2)
(3) $x_{34} + x_{36} - x_{13} - x_{43} = 0$ (node 3)
(4) $x_{42} + x_{43} + x_{45} + x_{46} + x_{47} - x_{14} - x_{24} - x_{34} - x_{54} - x_{64} = 0$ (node 4)
(5) $x_{52} + x_{54} + x_{57} - x_{25} - x_{45} = 0$ (node 5)
(6) $x_{64} + x_{67} - x_{36} - x_{46} = 0$ (node 6)
(7) $x_{71} - x_{47} - x_{57} - x_{67} = 0$ (node 7)
Capacity for each arc:
(8) $x_{12} \le 4$ (14) $x_{36} \le 6$ (20) $x_{52} \le 3$

(9)	$x_{13} \leq 3$	(15)	$x_{42} \leq 3$	(21)	$x_{54} \leq 4$
(10)	$x_{14} \leq 4$	(16)	$x_{43} \leq 5$	(22)	$x_{57} \leq 2$
(11)	$x_{24} \leq 2$	(17)	$x_{45} \leq 3$	(23)	$x_{64} \leq 1$
(12)	$x_{25} \leq 3$	(18)	$x_{46} \leq 1$	(24)	$x_{67} \leq 5$
(13)	$x_{34} \leq 3$	(19)	$x_{47} \leq 3$		

Nonnegativity:
All $x_{ij} \geq 0$

The LP was solved using *The Management Scientist*. Two solutions are given below.

Objective Function Value = 10.000

Variable	Solution 1	Solution 2
x12	3.000	3.000
x13	3.000	3.000
x14	4.000	4.000
x24	1.000	1.000
x25	2.000	2.000
x36	5.000	4.000
x43	2.000	1.000
x46	0.000	1.000
x47	3.000	3.000
x57	2.000	2.000
x67	5.000	5.000
x71	10.000	10.000

PTS: 1 TOP: Maximal flow problem

20. A foreman is trying to assign crews to produce the maximum number of parts per hour of a certain product. He has three crews and four possible work centers. The estimated number of parts per hour for each crew at each work center is summarized below. Solve for the optimal assignment of crews to work centers.

	Work Center			
	WC1	WC2	WC3	WC4
Crew A	15	20	18	30
Crew B	20	22	26	30
Crew C	25	26	27	30

ANS:
OBJECTIVE FUNCTION VALUE = 82.000

VARIABLE	VALUE	REDUCED COST
A_{A1}	0.000	12.000
A_{A2}	0.000	0.000
A_{A3}	0.000	0.000
A_{A4}	1.000	1.000
A_{B1}	0.000	2.000
A_{B2}	0.000	4.000
A_{B3}	1.000	0.000
A_{B4}	0.000	0.000
A_{C1}	0.000	12.000
A_{C2}	1.000	0.000

A_{C3}	0.000	0.000
A_{C4}	0.000	1.000
A_{D1}	1.000	2.000
A_{D2}	0.000	4.000
A_{D3}	0.000	0.000
A_{D4}	0.000	0.000

CONSTRAINT	SLACK/SURPLUS	DUAL PRICE
1	0.000	18.000
2	0.000	23.000
3	0.000	24.000
4	0.000	–1.000
5	0.000	1.000
6	0.000	2.000
7	0.000	3.000
8	0.000	12.000

An optimal solution is:

Crew	Work Center	Parts/Hour
Crew A	WC4	30
Crew B	WC3	26
Crew C	WC2	26
----------	WC1	---
	Total Parts Per Hour	= 82

PTS: 1 TOP: Assignment problem

21. A plant manager for a sporting goods manufacturer is in charge of assigning the manufacture of four new aluminum products to four different departments. Because of varying expertise and workloads, the different departments can produce the new products at various rates. If only one product is to be produced by each department and the daily output rates are given in the table below, which department should manufacture which product to maximize total daily product output? (Note: Department 1 does not have the facilities to produce golf clubs.)

Department	Baseball Bats	Tennis Rackets	Golf Clubs	Racquetball Rackets
1	100	60	X	80
2	100	80	140	100
3	110	75	150	120
4	85	50	100	75

Formulate this assignment problem as a linear program.

ANS:

x_i represent all possible combinations of departments and products
For example: x_1 = 1 if Department 1 is assigned baseball bats; = 0 otherwise
x_2 = 1 if Department 1 is assigned tennis rackets; = 0 otherwise
x_5 = 1 if Department 2 is assigned baseball bats; = 0 otherwise
x_{15} = 1 if Department 4 is assigned golf clubs; = 0 otherwise
(Note: x_3 is not used because Dept.1/golf clubs is not a feasible assignment)

$$\text{Min } Z = 100x_1+60x_2+80x_4+100x_5+80x_6+140x_7+100x_8+110x_9 +75x_{10}+150x_{11}+120x_{12}+85x_{13}+50x_{14}+100x_{15}+75x_{16}$$

S.T.

$$x_1 + x_2 + x_4 = 1$$
$$x_5 + x_6 + x_7 + x_8 = 1$$
$$x_9 + x_{10} + x_{11} + x_{12} = 1$$
$$x_{13} + x_{14} + x_{15} + x_{16} = 1$$
$$x_1 + x_5 + x_9 + x_{13} = 1$$
$$x_2 + x_6 + x_{10} + x_{14} = 1$$
$$+ x_7 + x_{11} + x_{15} = 1$$
$$x_4 + x_8 + x_{12} + x_{16} = 1$$

PTS: 1 TOP: Assignment problem